# Incremental Algorithms for Inter-procedural Analysis of Safety Properties[*]

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**Abstract**

Automaton-based static program analysis has proved to be an effective tool for bug finding. Current tools generally re-analyze a program from scratch in response to a change in the code, which can result in much duplicated effort. We present an inter-procedural algorithm that analyzes *incrementally* in response to program changes and present experiments for a null-pointer dereference analysis. It shows a substantial speed-up over re-analysis from scratch, with a manageable amount of disk space used to store information between analysis runs.

## 1 Introduction

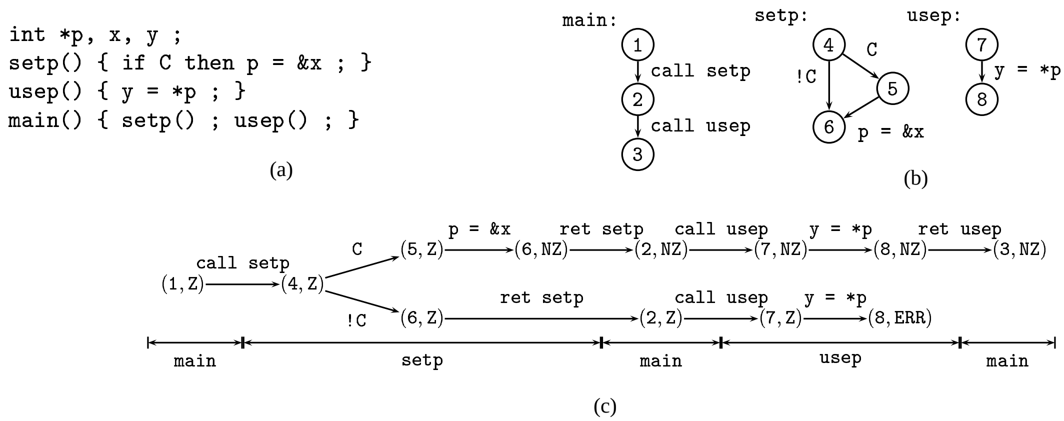
Tools based on model checking with automaton specifications have been very effective at finding important bugs such as buffer overflows, memory safety violations, and violations of locking and security policies. Static analysis tools such as MC/Coverity [10] and Uno [13], and model checking tools such as SLAM [2] are based on inter-procedural algorithms for propagating dataflow information [20, 9, 1, 3]. These algorithms perform a reachability analysis that always starts from scratch. For small program changes—which often have only a localized effect on the analysis—this can be inefficient.

Our main contribution is to present the first, to our knowledge, incremental algorithms for safety analysis of recursive state machines. We demonstrate how these algorithms can be used to obtain simple—yet general and precise—incremental automaton-based program analyses. We give two such algorithms: one that operates in the forward direction from the initial states and another that operates "inside-out" from the locations of the program changes. These have different tradeoffs, as is true of forward and backward algorithms for model checking safety properties. The key to both algorithms is a data structure called a *derivation graph*, which records the analysis process. In response to a program change, the algorithms re-check derivations recorded in this graph, pruning those that have been invalidated due to the change and adding new ones. This repair process results in a new derivation graph, which is stored on disk and used for the following increment.

A prototype implementation of these algorithms has been made for the Orion static analyzer for C and C++ programs [6]. Our measurements show significant speedup for both algorithms when compared with a non-incremental version. This comes at the expense of a manageable increase in disk usage for storing information between analysis runs. We expect our algorithms to be applicable to many current program analysis tools.

The algorithms we present are incremental forms of a standard model checking algorithm. As such, their verification result is identical to that of the original algorithm. The implementation is part of a static analysis tool that checks

[*] This is an expanded and revised version of a paper presented at CAV 2005 [5].



```
int *p, x, y ;
setp() { if C then p = &x ; }
usep() { y = *p ; }
main() { setp() ; usep() ; }
```

Figure 1: (a) An example program, (b) its function CFGs, and (c) the derivation graph.

an abstraction of C or C++ code. Thus, there is some imprecision in its results: the analysis may report false errors and miss real ones. However, the incremental algorithms produce reports with the same precision as the non-incremental algorithm.

Incremental model checking may have benefits beyond speeding up analysis. One direction is to trade the speed gain from incremental analysis for higher precision in order to reduce the number of false errors reported. Another direction is to integrate a fine-grained incremental model checker into a program development environment, so that program errors are caught immediately, as has been suggested for testing [22]. A third direction is to use an incremental model checker to enable correct-by-construction development, as suggested by Dijkstra [7]. In this scenario, instead of applying model checking after a program is written, an incremental model checker can maintain and update a proof of correctness during program development. Our work is only a first step towards realizing the full potential of these possibilities.

Full proofs of all theorems are presented in Appendix A.

## 1.1 An example

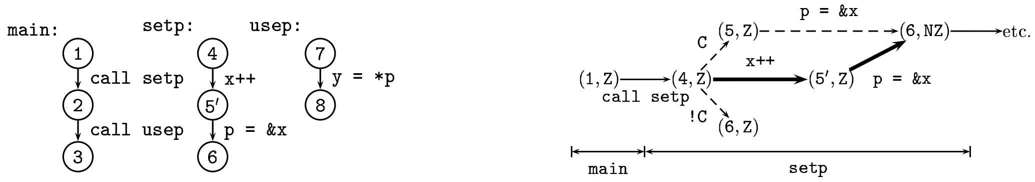
The input to the basic algorithm is a program, described by a collection of control-flow graphs (CFGs), and a checking automaton. The nodes of a CFG represent control locations, while edges are labeled either with simple assignment statements, (side-effect free) assertions, or function calls. In the model checking view, the (possibly non-deterministic) checking automaton "runs" over matched call-return paths in this collection of CFGs, flagging a potential program error whenever the current run enters an error state.

The basic model checking algorithm works by building, on-the-fly, a "synchronous product" graph of the collective CFGs with the automaton. At a function call edge, this product is constructed by consulting a summary cache of entry-exit automaton state pairs for the function. Using this cache has two consequences: it prevents infinite looping when following recursive calls and it exploits the hierarchical function call structure, so that function code is not unnecessarily re-examined.

The key to the incremental version of the algorithm is to observe that the process of forming the synchronous product can be recorded as a derivation graph. After a small change to the CFGs, it is likely that most of the process of forming the synchronous product is a repetition of the earlier effort. By storing the previous graph, this repetitive calculation can be avoided by checking those portions that may have been affected by the change, updating derivations only when necessary.

To illustrate these ideas, consider the program in Fig. 1(a). The correctness property we are interested in is whether the global pointer p is initialized to a non-null value before being dereferenced. A simple automaton (not shown) to check for violations of this property has three states: Z, indicating p may be null; NZ, indicating p is not null; and the error state ERR indicating p is dereferenced when it may be null.

Figures 1(b) and 1(c) show, respectively, the CFGs for this program and the resulting derivation graph (in this case a tree). Each derivation graph node is the combination of a CFG node and an automaton state. If condition C holds on entry to setp (the upper branch from the state $(4, \mathtt{Z})$ in setp), the function returns to main with the automaton



(a) (b)

Figure 2: (a) The revised CFGs and (b) a portion of the incremental derivation graph.

state `NZ`, and execution proceeds normally to termination. If `C` does not hold (the lower branch), `setp` returns to `main` with the automaton state `Z`. On the statement "`y = *p`", the automaton moves to the state `ERR` and an error is reported in `usep`.

The incremental algorithm operates on the derivation graph data structure. Besides this graph, its input consists of additions, deletions, and modifications to CFG edges. The basic idea is simple: inspect each derivation step to determine whether it is affected by a change; if so, remove the derivation and re-check the graph from the affected point until a previously explored state is encountered.

For our example, consider the revision obtained by replacing the body of `setp()` by "`x++; p = &x;`". The new CFGs are shown in Fig. 2(a). Figure 2(b) shows the incremental effect on the derivation graph. The removal of the `if` statement has the effect of removing the conditional branch edges (dashed) from the graph, making the previous error state unreachable. The addition of `x++` has the effect of adding the state $(\mathtt{5}', \mathtt{Z})$ and two edges (bold) to the graph. After processing these edges, we get to state $(\mathtt{6}, \mathtt{NZ})$, which is identical to the corresponding state in the previous analysis. At this point, we should be able to terminate the analysis. This is a simplified picture: our algorithms actually operate somewhat differently. In particular, the backward algorithm will also inspect the derivation graph in `main`, but not that for `usep`.

# 2 The Full Analysis Algorithm

A program is given as a set $\mathcal{F}$ of functions, with a distinguished initial function, $main$. Each function is represented by a CFG, which is a tuple $(N, \Sigma, E)$. Here, $N$ is a finite set of *control locations* containing the distinguished locations $\downarrow$ (entry) and $\uparrow$ (exit); $\Sigma$ is a set of *(simple) program statements* (assignments and assertions); and $E$ is the set of *edges*. Let $\Sigma'$ be $\Sigma$ together with call statements $\{call(f) \mid f \in \mathcal{F}\}$. $E$ is a subset of $(N \backslash \{\uparrow\}) \times \Sigma' \times N$. We require that there are no calls to functions outside $\mathcal{F}$. For simplicity of exposition, we do not represent function call arguments and return values, or variables and their scoping rules. The implementation takes each of these features into consideration.

Next we define the executions of a program. A *position* is a pair $(f, n)$, where $f$ is a function and $n$ is a node in (the CFG for) $f$. A *(global) program state* is a sequence $(f_1, n_1) \cdots (f_k, n_k)$ of positions, representing a point during execution where control resides at position $(f_k, n_k)$ and $(f_1, n_1) \cdots (f_{k-1}, n_{k-1})$ is the stack of return locations that is in effect at this point. We define a labeled transition system on program states, as follows.

1. $(f_1, n_1) \cdots (f_k, n_k) \xrightarrow{a} (f_1, n_1) \cdots (f_k, n'_k)$ iff $(n_k, a, n'_k)$ is an edge in $f_k$ and $a$ is not a call

2. $(f_1, n_1) \cdots (f_k, n_k) \rightarrow (f_1, n_1) \cdots (f_k, n'_k)(f', \downarrow)$ iff $(n_k, call(f'), n'_k)$ is an edge in $f_k$

3. $(f_1, n_1) \cdots (f_{k-1}, n_{k-1})(f_k, \uparrow) \rightarrow (f_1, n_1) \cdots (f_{k-1}, n_{k-1})$

An *execution* is a finite path in this transition system that begins with the program state $(main, \downarrow)$, consisting of just the initial position. Such an execution generates a *trace* consisting of the sequence of labels (which are program statements) along it. Note that this is the definition of a recursive state machine [1, 3], restricted to the case of finite executions.

Analysis properties are represented by (non-deterministic, error detecting) automata with $\Sigma$ as input alphabet. An analysis automaton is given by a tuple $(Q, \hat{Q}, \Delta, F)$, where $Q$ is a set of *(automaton) states*, $\hat{Q} \subseteq Q$ is a set of *initial states*, $\Delta \subseteq Q \times \Sigma \times Q$, is a *transition relation*, and $F \subseteq Q$ is a set of *rejecting states*. A *run* of the automaton on a trace is defined in the standard way. A *rejecting run* is a run that includes a rejecting state. Note that in this simplified presentation, the set $\Sigma$ of program statements does not include function calls and returns, and hence the automata

```
ADD-TO-WORKSET($c$)
 1  if $c$ is not marked then
 2    $workset \leftarrow workset \cup \{c\}$
 3    mark $c$

FOLLOW-EDGE($c, e$)
 1  // c=(f,n,r,q), e=(n,a,n′)
 2  if $a = call(f')$ then
 3    // use summaries; do book-keeping
 4    ADD-TO-WORKSET($(f', \downarrow, q, q)$)
 5    Add $c$ to $call\text{-}sites(f')$
 6    for $q' : \langle q, q' \rangle \in summary(f')$ do
 7      ADD-TO-WORKSET($(f, n', r, q')$)
 8  else
 9    // follow automaton transition
10    for $q' : (q, a, q') \in \Delta$ do
11      ADD-TO-WORKSET($(f, n', r, q')$)
```

```
STEP($c = (f, n, r, q)$)
 1  if $q \in F$ then
 2    REPORT-ERROR($c$)
 3  if $n = \uparrow$ then
 4    // add a summary pair
 5    Add $\langle r, q \rangle$ to $summary(f)$
 6    $workset \leftarrow workset \cup call\text{-}sites(f)$
 7  else
 8    // follow a CFG edge
 9    for $e \in edges(n)$ do
10      FOLLOW-EDGE($c, e$)

ANALYZE
 1  $workset \leftarrow \{(main, \downarrow, q, q) \mid q \in \hat{Q}\}$
 2  while $workset \neq \emptyset$ do
 3    Remove some $c \in workset$
 4    STEP($c$)
```

Figure 3: Pseudo-code for the Full Algorithm.

cannot refer to them. In the implementation, transitions that represent function calls and returns (rules 2 and 3 above) carry special labels, and the error detecting automaton can react to them by changing its state, e.g. to perform checks of the arguments passed to a function, or the value returned by it.

We emphasize that an automaton operates on the *syntax* of the program; the relationship with the semantics is up to the automaton writer. For instance, one might define an under-approximate automaton, so that any error reported by the automaton check is a real program error, but it might not catch all real errors. It is more common to define an over-approximate automaton, so that errors reported are not necessarily real ones, but the checked property holds if the automaton does not find any errors.

The pseudo-code for the from-scratch analysis algorithm (Full) is shown in Fig. 3. It keeps *global configurations* in a work-set; each configuration is a tuple $(f, n, r, q)$, where $(f, n)$ is a position and $r, q$ are automaton states. The presence of such a configuration in the work-set indicates that it is possible for a run of the automaton to reach position $(f, n)$ in automaton state $q$ as a result of entering $f$ with automaton state $r$ (the "root" state). In addition, the algorithm keeps a set of summaries for each function, which are entry-exit automaton state pairs, and a set of known call-sites, which are configurations from which the function is called. ANALYZE repeatedly chooses a configuration from the work-set and calls STEP to generate its successors. In STEP, if the automaton is in an error state, a potential error is reported. (In an implementation, the REPORT-ERROR procedure may also do additional work to check if the error is semantically possible.)

Much of the work is done in the FOLLOW-EDGE procedure. For a non-call statement, the procedure follows the automaton transition relation (Line 10). For a function call, the procedure looks up the summary table to determine successor states (Line 6). If there is no available summary, registering the current configuration in $call\text{-}sites(f')$ and creating a new entry configuration for $f'$ ensures that a summary entry will be created later, at which point this configuration is re-examined (Line 6 in STEP). We assume that visited configurations are kept in a suitable data structure (e.g., a hash-table).

**Theorem 1.** *The Full algorithm reports an error at a configuration $(f, n, r, q)$, for some $r, q$, if and only if there is a program execution ending at a position $(f, n)$, labeled with trace $t$, such that the automaton has a rejecting run on $t$.*

## 3 A First Incremental Algorithm: IncrFwd

**Input:** A textual program change can be reflected in the CFGs as the addition, deletion, or modification of control-flow edges. It can also result in the redefinition of the number and types of variables. Our incremental algorithms

```
ADD-TO-WORKSET($c$)
if $c$ is not marked then
    $workset \leftarrow workset \cup \{c\}$
    mark $c$

CHECK-EDGE($e = c \vdash_a c'$)
// c = (f,n,r,q), c′ = (f,n′,r,q′)
if $(n, a, n')$ is a deleted edge then
    skip
elseif $a = call(f')$ then
    // use stored summaries
    ADD-TO-WORKSET($(f', \downarrow, q, q)$)
    Add $c$ to $call\text{-}sites(f')$
    if $\langle q, q' \rangle$ is marked in $f'$ then
        ADD-TO-WORKSET($c'$); mark $e$
else
    ADD-TO-WORKSET($c'$); mark $e$

CHECK-STEP($c = (f, n, r, q)$)
if $n = \uparrow$ then
    Mark the summary $\langle r, q \rangle$ in $f$
    $workset \leftarrow workset \cup call\text{-}sites(f)$
else
    for each deriv. edge $e$ from $c$ do
        CHECK-EDGE($e$)

CHECK-DERIVATIONS($Fns$)
for $f \in Fns$ do
    Unmark $f$'s configs, edges,
        summaries, and call sites that
        originate in $Fns$
$workset \leftarrow$ EXT-INITS($Fns$)
while $workset \neq \emptyset$ do
    Choose and remove $c \in workset$
    CHECK-STEP($c$)
Remove unmarked elements
```

Figure 4: The IncrFwd algorithm for Deletions.

expect as input CFG changes, and repair the derivation graph accordingly. Changes to types and variables correspond to automaton modifications. The algorithm can be easily modified for the situation where the property—and not the program—changes, since we are maintaining their joint (i.e., product) derivation graph.

**Data Structure:** The incremental algorithm records a derivation relation on configurations. This is done in the procedure FOLLOW-EDGE: whenever a new configuration of the form $(f, n', r, q')$ is added after processing a configuration $(f, n, r, q)$ and an edge $a$, a derivation edge $(f, n, r, q) \vdash_a (f, n', r, q')$ is recorded. This results in a labeled and directed derivation graph. Notice that the derivation graph can be viewed also as a tableau proof that justifies either the presence or absence of reachable error states.

Given as input a set of changes to the CFGs and a derivation graph, the incremental algorithm first processes all the modifications, then the deletions, and finally the additions. This order avoids excess work where new configurations are added only to be retracted later due to CFG deletions.

**Modifications:** For an edge $e = (n, a, n')$ modified to $e' = (n, b, n')$ in function $f$, if each derivation of the form $(f, n, r, q) \vdash_a (f, n', r, q')$ holds also for the new statement $b$—which is checked by code similar to that in FOLLOW-EDGE— there is no need to adjust the derivation graph. Otherwise, the modification is handled as the deletion of edge $e$ and the addition of $e'$.

**Additions:** For a new edge $e = (n, a, n')$ in the CFG of $f$, FOLLOW-EDGE is applied to all configurations of the form $c = (f, n, r, q)$, for some $r, q$, that are present in the current graph. Consequently, any newly generated configurations are processed as in the full algorithm.

**Deletions:** Deletion is the non-trivial case. Informally, the idea is to check all of the recorded derivation steps, disconnecting those that are based on deleted edges. The forward-traversing deletion algorithm (IncrFwd) is shown in Fig. 4. The entry point is the procedure CHECK-DERIVATIONS, which is called with the full set of functions, $\mathcal{F}$. The auxiliary function EXT-INITS($F$) returns the set of entry configurations for functions in $F$ that arise from a call outside $F$. The initial configurations for $main$ are considered to have external call sites. This gives a checking version of the full analysis algorithm. Checking an existing derivation graph can be expected to be faster than regenerating it from scratch with the full algorithm. The savings can be quite significant if the automaton transitions $\Delta$ are computed on-the-fly—notice that the algorithm does not re-compute $\Delta$. The similarity between the Full and IncrFwd algorithms can be formalized in the following theorem.

**Theorem 2.** *The derivation graph resulting from the IncrFwd algorithm is the same as the graph generated by the Full analysis algorithm on the modified CFGs.*

```
RETRACE($c = (f, n, r, q)$)
if $c$ is not marked then
  return $false$
elseif $f = main$ then
  return $true$
else
  return ($\exists c' = (f', n', r', r)$:
      $c' \in call\text{-}sites(f) \wedge$ RETRACE($c'$))
```

```
INCR-BACK()
// bottom-up repair
for each SCC $C$ (in reverse
    topological order) do
  if AFFECTED($C$) then
    CHECK-DERIVATIONS($C$)
// remove unreachable errors
for each error configuration $c$ do
  if (not RETRACE($c$)) then
    unmark $c$
```

Figure 5: The IncrBack algorithm for Deletions.

# 4 A Second Incremental Algorithm: IncrBack

The IncrFwd algorithm checks derivations in a forward traversal. This might result in unnecessary work: if only function $g$ is modified, functions that are not on any call path that includes $g$ are not affected, and do not need to be checked. Moreover, if the change to $g$ does not affect its summary information, even its callers do not need to be checked. Such situations can be detected with an "inside-out" algorithm, based on the maximal strongly connected component (SCC) decomposition of the function call graph. (A non-trivial, maximal SCC in the call graph represents a set of mutually recursive functions.)

The effect of a CFG edge deletion from a function $f$ propagates both upward and downward in the call graph. Since some summary pairs for $f$ may no longer be valid, derivations in $f$'s callers might be invalidated. In the other direction, for a function called by $f$, some of its entry configurations might now be unreachable.

The SCC-based algorithm (IncrBack) is shown in Fig. 5. It works bottom-up on the SCC decomposition, checking first the lowest (in topological order) SCC that is affected. The function AFFECTED($C$) checks whether a function in $C$ is modified, or whether summaries for any external function called from $C$ have been invalidated. For each SCC $C$, one can inductively assume that summaries for functions below $C$ are valid. Hence, it is only necessary to examine functions in $C$. This is done by the same CHECK-DERIVATIONS procedure as in Fig. 4, only now applied to a single SCC instead of the full program. Note that CHECK-DERIVATIONS initially invalidates summaries in $C$ that cannot be justified by calls outside $C$.

This process can result in over-approximate reachability information. Consider a scenario where $f$ calls $g$. Now suppose that $f$ is modified. The algorithm repairs derivations in $f$, but does not touch $g$. However, derivations in $f$ representing calls to $g$ might have been deleted, making corresponding entry configurations for $g$ unreachable. To avoid reporting spurious errors resulting from this over-approximation, the (nondeterministic) RETRACE procedure re-determines reachability for all error configurations.

**Theorem 3.** *The derivation graph after the IncrBack algorithm is an over-approximation of the graph generated by the full analysis algorithm on the modified CFGs, but has the same set of error configurations.*

# 5 Complexity and Optimality

The non-incremental algorithm takes time and space linear in the product of the size of the automaton and the size of the collective control-flow graphs. Algorithms with better bounds have been developed [1, 3], but these are based on knowing in advance the number of exit configurations of a function; this is impossible for an on-the-fly state exploration algorithm.

From their similarity to the non-incremental algorithm, it follows that the incremental algorithms cannot do more work than the non-incremental one, so they have the same worst-case bound. However, worst-case bounds are not particularly appropriate, since incremental algorithms try to optimize for the common case. Ramalingam and Reps [19, 18] propose to analyze performance in terms of a quantity $||\delta||$, which represents the difference in reachability after a change. They show that any "local" incremental algorithm like ours has worst-case inputs where the work cannot be

|  | Lines<br>of code | Reachable<br>functions | No. ptrs<br>analyzed | Full analysis<br>time (s) | Average speedup<br>IncrFwd | <br>IncrBack | Incr. data<br>(KB) |
|---|---|---|---|---|---|---|---|
| sendmail | 47,651 | 336 | 3 | 33.75 | 1.6 | **8.6** | 73.72 |
| spin | 16,540 | 348 | 6 | 24.91 | 1.3 | **10.1** | 91.61 |
| spin:tl | 2,569 | 93 | 2 | 1.09 | 1.3 | **8.0** | 7.01 |
| guievict | 4,545 | 115 | 1 | 0.60 | 1.4 | **5.9** | 3.54 |
| rocks | 4,619 | 134 | 1 | 0.66 | 1.3 | **4.3** | 4.36 |

Table 1: Experimental results.

bounded by a function of $||\delta||$ alone. At present, the precise complexity of incremental reachability remains an open question [12].

# 6 Implementation and Experiments

We have implemented the Full, IncrFwd, and IncrBack algorithms in the Orion static analyzer. In the implementation, we take a function as the unit of change. This is done for a number of reasons. It is quite difficult, without additional machinery (such as an incremental parser), to identify changes of finer granularity. It also fits well into the normal program development process. Furthermore, functions scale well as a unit of modification—as the size of a program increases, the relative size of individual functions decreases. In the case of large programs, attempting to identify changes at the CFG or parse tree level may not lead to significant gains.

We present data on five open source applications: sendmail, the model checker spin, spin:tl (spin's temporal logic utility), guievict (an X-Windows process migration tool) and rocks (a reliable sockets utility). We perform an interprocedural program analysis from a single entry function, checking whether global pointers are set before being dereferenced. For sendmail and spin, this analysis is run for a small subset of the program's global pointers in order to reduce the time necessary for the experiments. We simulate the incremental development of code as follows. For each function $f$ in the program, we run the incremental analysis on the program with $f$ removed (i.e., replaced with an empty stub). Then we insert $f$ and run the incremental analysis. The time taken for this incremental analysis is compared with a full analysis of the program with $f$. We thus have one analysis run for each function in the program; each run represents an incremental analysis for the modification of a single function (in this case, replacing an empty stub with the actual body of the function).

This experiment exercises both the addition and deletion algorithms: the modification of a function is equivalent to deleting and reinserting a call edge at each of its call sites; if the function summary changes, derivations based on the old summary are deleted and new derivations are generated based on the newly available, now-accurate summary.

The experimental results are shown in Table 1. The overall average speedup for IncrBack is 8.2; the average for IncrFwd is 1.4. IncrFwd improves on Full essentially by "caching" state data between analysis runs. This caching behavior is able to provide modest performance increases, but the average-case performance of the algorithm is unbounded in $||\delta||$. IncrBack is able to improve on the performance of IncrFwd by skipping large portions of the derivation graph, when possible, and using the call graph structure of the program to minimize its workload. This intuition is confirmed by the experimental results.

To better illustrate the performance characteristics of IncrBack, Fig. 6 plots the speedups for each analysis run in terms of percentiles. For each percentage $x$, we plot the minimum speedup for the best-performing $x\%$ of analysis runs. For example, 50% of analysis runs overall showed a speedup of at least 7.5 (i.e., 7.5 is the median speedup). The legend shows the number of analysis runs (i.e., the number of (statically) reachable functions) for each benchmark. The data on the horizontal axis is plotted in uniform intervals of length $100/N$ for each benchmark. The plateaus evident in the plots for spin, spin:tl, and rocks represent clustering of data values (possibly due to rounding) rather than a sparsity of data points.

There was quite a bit of variation between benchmarks: over 50% of runs on spin showed a speedup of at least 12.4, while the maximum speedup for rocks was only 5.5. It is likely that larger programs will exhibit higher speedups in general. We observed no strong correlation between the speedup for a function and its size, its depth in the call graph, or its number of callers and callees (see Appendix B).

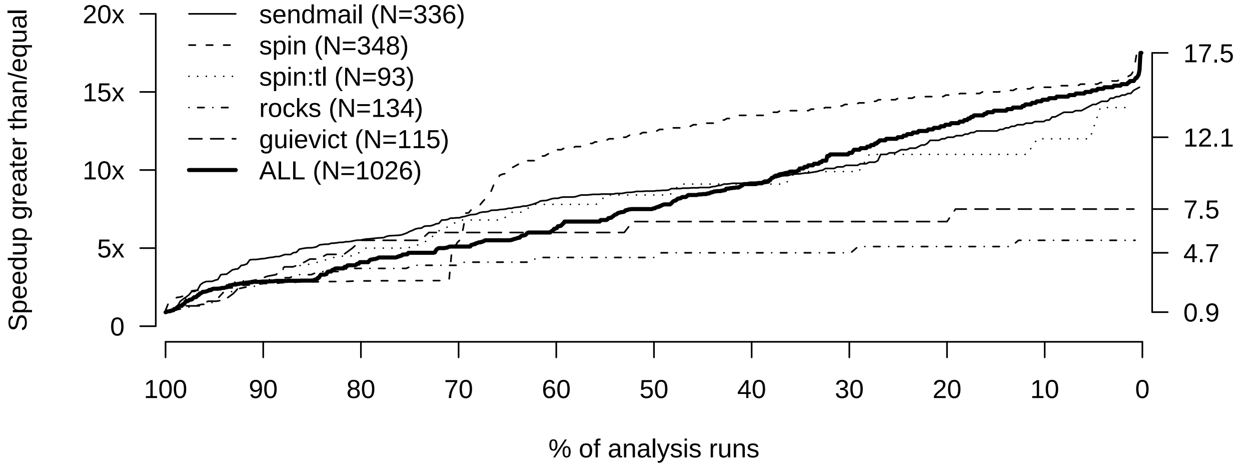


Figure 6: Distribution of speedups for IncrBack, with quantiles for all at right.

The tests we describe here are conservative in the sense that we only analyze functions that are reachable from a single distinguished entry function. Preliminary tests on all of the entry functions in guievict show an average speedup of 11.3 (instead of 5.9)—since many functions are unconnected in the call graph to the modified function, the full algorithm does much unnecessary work.

These tests concentrate on changes that are quite small with respect to the total size of the program. We hypothesize that changes on the order of a single function are a reasonable model of how the analysis would be applied in a development scenario. However, we have also run tests in which 10-50% of the functions in the program are modified in each increment. In these tests, IncrBack showed a more modest average speedup of 2.5, with larger speedups for smaller incremental changes.

Table 1 also shows the size of the incremental data stored after the re-run of the incremental analysis, on the complete program. This data may be written to disk, taking time proportional to the size. In an interactive setting, this time can be considered irrelevant: the user can begin inspecting errors while the tool performs I/O.

## 7 Related Work and Conclusions

The approach of automaton-based model checking of push-down systems [25, 1, 3] has contributed algorithms for program analysis that are conceptually simple and powerful. We have developed incremental versions of these algorithms and shown that this approach leads to incremental dataflow algorithms that are simple yet precise and general. The algorithms lend themselves to simple implementations showing excellent experimental results: a factor of 8.2 on average for IncrBack, at the cost of a manageable overhead in storage. To the best of our knowledge, the algorithms we propose are the first for inter-procedural, automaton-based static analysis.

There is a strong similarity between the behavior of our checking procedure and tracing methods for garbage collection [16] (cf. [29]). A key difference is the pushdown nature of the derivation graphs, which has no analogy in garbage collection.

Incremental data flow analysis has been studied extensively. Existing data flow algorithms are not directly applicable to model checking, because they either compute less precise answers than their from-scratch counterparts; are applicable only to restricted classes of graphs, such as reducible flow-graphs; or concern specific analyses, such as points-to analysis [31, 30, 28] (cf. the excellent survey by Ramalingam and Reps [17]). Sittampalam et al. [26] suggest an approach to incremental analysis tied to program transformation (cf. [15]). Since analyses are specified on the abstract syntax tree, the technique only applies to idealized Pascal-like languages. The SCC decomposition has been applied to the flow graphs of individual functions to speed up analysis by Horwitz et al. [14] and Marlowe and Ryder [21].

Perhaps most closely related to our work is the research on the incremental evaluation of logic programs by Saha and Ramakrishnan [23, 24]. Their support graphs play the same role as derivation graphs in our work. The techniques used for updating these graphs are reminiscent of Doyle's truth maintenance system [8]. While inter-procedural analysis is readily encoded as a logic program (cf. [1]), we suspect that it may be hard to recover optimizations such as the SCC-based method. Previous algorithms for incremental model checking [27, 11] do not handle either program hierarchy or recursion, working instead with a flat state space. Some tools (e.g., Uno [13], MOPS [4]) pre-compute per-file information; however, the interprocedural analysis is still conducted from scratch.

The Orion tool in which the algorithms are implemented is aimed at producing error reports with a low false-positives ratio. In this context, it seems especially attractive to devote the time gained by incrementalization towards a further improvement of this ratio, especially for inter-procedural analysis.

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# A Proofs

In the proofs, we refer to a sequence of transitions in the global transition graph as a *path*; an execution is a path starting with the initial global state. We write a path as $x_0; a_0; x_1; a_1; \ldots; x_n$ where the $x_i$ are global states, the $a_i$ are elements of $\Sigma \cup \{\epsilon\}$, and each triple $(x_i, a_i, x_{i+1})$ is a valid transition. The length of a path is the number of transitions on it ($n$ for the path above). For a path as above, $a_0; a_1; \ldots; a_{n-1}$ is the trace of actions along it. Since an automaton runs on such traces, we can merge a run $q_0; a_0; q_1; a_1; \ldots; q_n$ of the automaton along this trace into the path itself, thinking of it as a run of the automaton on the path. This is written as $(x_0, q_0); a_0; (x_1, q_1); a_1; \ldots; (x_n, q_n)$.

A configuration $c$ is said to be *generated* during an execution of an algorithm iff it is added to the algorithm's $\mathit{workset}$ at any point during that execution. For example, in the case of the Full Algorithm (Figure 3), this means that it is added to $\mathit{workset}$ by line 1 of ANALYZE, line 6 of STEP, or line 2 of ADD-TO-WORKSET. Similarly, a configuration is said to be *processed* iff it is chosen and removed from $\mathit{workset}$, e.g., by line 3 of ANALYZE.

**Lemma 1.** *For each configuration $(f, n, p, q)$ that is generated by the algorithm, there is a program execution to a global state of the form $x(f, n)$, for some $x$, and a run of the automaton on this execution that is in state $q$ at the end, and in state $p$ at the last call to $f$.*

*Proof.* The proof is by induction on the number of algorithm steps. We strengthen this claim by adding the following: for any function $f$, and every pair $\langle q, q' \rangle$ in the summary for $f$, there is a path $\pi$ starting with a global state $x(f, \downarrow)$ and ending at state $x(f, \uparrow)$ (for some prefix $x$) and a run of the automaton on this path starting at state $q$ and ending at state $q'$.

Basis (0 steps): the claim is true by the initialization of the workset, and as all summaries are initially empty.

Induction (k+1 steps): assume that the claim holds after $k$ steps and consider the configurations and summaries generated in the $k + 1$'st call to the STEP function. Let $c = (f, n, p, q)$ be the configuration examined at this stage. By the IH, there is an execution $\pi$ whose final state has the form $x(f, n)$, and a run $\rho$ on $\pi$ where $q$ is the end state and $p$ is the state at the last call to $f$ on $\pi$.

(i) if $n =\uparrow$, the pair $\langle p, q \rangle$ is added to the summary for $f$. The suffix of $\rho$ beginning at the last call to $f$ defines a path and a run on it from $p$ to $q$, which validates the IH for the new summary pair. The call sites added to the workset are not newly generated configurations, so the IH holds for those.

(ii) now suppose that $n \neq\uparrow$. For a configuration generated in following an edge $e = (n, a, n')$ from $n$, there are two possibilities. Write $\rho$ as $\rho'; a; (x(f, n), q)$.

(a) $a = \mathit{call}(f')$. Two configurations are generated. Firstly, the configuration $(f', \downarrow, q, q)$. Consider the path $\pi; \epsilon; (x(f, n')(f', \downarrow))$ defined by rule 2 in Section 2. The run $\rho'; a; (x(f, n), q); \epsilon; (x(f, n')(f', \downarrow), q)$ over this path justifies the new configuration.

In addition, a configuration $(f, n', p, q')$ might be added, where $\langle q, q' \rangle$ is a summary pair for $f'$. By IH, the summary pair is associated with a run of the form $(y(f', \downarrow), q) \cdot \sigma[y] \cdot (y(f', \uparrow), q')$, for some $y$, where each global state in $\sigma[y]$ has prefix $y$. The stack nature of the global transition graph implies that this input-output behavior is actually valid for any $y$. Consider the run obtained from $\rho$ where $f'$ is called, does the sequence of actions specified in the trace for $\sigma$, and returns to $f$: $\rho; \epsilon; (x(f, n')(f', \downarrow), q) \cdot \sigma[x(f, n')] \cdot (x(f, n')(f', \uparrow), q'); \epsilon; (x(f, n'), q')$. This run justifies the configuration $(f, n', p, q')$.

(b) $a$ is a non-call edge. Then a configuration $(f, n', p, q')$ can be added, where $q' \in \Delta(q, a)$. Extend the run $\rho$ to $\rho; a; (x(f, n'), q')$, which justifies the new configuration. □

We need an auxiliary lemma that is used in Lemma 3 below. It is essentially a termination argument.

**Lemma 2.** *When a configuration $c$ is generated during an execution of the Full Algorithm, then $c$ is processed during the same execution.*

*Proof.* First, observe that for any function $g$, the set $\mathit{call\text{-}sites}(g)$ cannot contain a configuration of the form $(x, \uparrow, y, z)$ (for any $x$, $y$, $z$). Namely, the only place where a configuration, say $(f, n, r, q)$, is added to a $\mathit{call\text{-}sites}$ set, say $\mathit{call\text{-}sites}(f')$, is on line 5 of FOLLOW-EDGE. In this case, the algorithm is considering an edge $e = (n, \mathit{call}(f'), n')$ for some $n'$. If $n =\uparrow$, then no such edge exists, by the definition of edges (beginning of Section 2).

Suppose that there is a configuration that is generated during an execution of the Full Algorithm, but which is never processed. Then it must be that the algorithm does not terminate, and executes line 3 of ANALYZE infinitely often. Since there are finitely many configurations, there must be a configuration, say $c$, that is chosen and removed infinitely often on that line. So $c$ must be infinitely often added to $workset$. Clearly, no configuration can be added more than once on line 1 of ANALYZE. Also, it cannot be added more than once on line 2 of ADD-TO-WORKSET, because it will get marked then, causing the test on line 1 of ADD-TO-WORKSET to fail the next time around. So $c$ must be infinitely often added on line 6 of STEP, which means $c \in \mathit{call\text{-}sites}(f)$ for some $f$. Because of the condition on line 3 of STEP, this implies that some configuration $d$ of the form $(x, \uparrow, y, z)$ is processed infinitely often, and hence $d$ must be generated infinitely often. But since a configuration of that form cannot be element of a $\mathit{call\text{-}sites}$ set, $d$ must be added infinitely often on line 1 of ANALYZE or on line 2 of ADD-TO-WORKSET. But that is impossible, for the reason given above. □

**Lemma 3.** *If there is an execution with final state $x(f, n)$, for some $x$, and a run of the automaton on this path that is in state $q$ at the end, and in state $p$ at the last call to $f$, then the configuration $(f, n, p, q)$ is generated by the algorithm.*

*Proof.* The proof is by induction on the number of transitions along the execution path.

Basis (0): The final state of the execution is an initial global state, so the claim holds by the initialization of the worklist.

Induction $(k + 1)$: Suppose that the claim holds for executions of length $k$. Consider a run on an execution of length $k + 1$. This has the form $((main, \downarrow), q_0); \ldots ; (x(f, n), q); a; (y, q')$, where $q_0$ is an initial automaton state. By the IH, the configuration $(f, n, p, q)$ is generated at some point by the algorithm, where $p$ is the automaton state at the last call to $f$. We have to show that a configuration appropriate to $(y, q')$ is generated as well. There are three possibilities.

(i) $a$ is not a call or return transition, so that $y = x(f, n')$, and $q' \in \Delta(q, a)$. When $(f, n, p, q)$ is processed, this case leads to the creation of $(f, n', p, q')$.

(ii) $a$ is a call transition, so that $y = x(f, n)(f', \downarrow)$ and $q' = q$. The processing of the transition creates the configuration $(f', \downarrow, q, q)$, as appropriate.

(iii) $a$ is a return step, so that $n = \uparrow$, $q' = q$ and $y = x$. I.e., the final state of the run is $(x, q)$. By the IH, the configuration $(f, \uparrow, p, q)$ must be generated. Hence, a summary pair $\langle p, q\rangle$ is added to the summary for $f$ when this configuration is processed. Suppose that $x$ has the form $x'(f', n')$. We have to show that $(f', n', p', q)$, for $p'$ being the automaton state at the last call to $f'$, is generated. Suppose that $x'(f', m)$ is the global state at the start of the last call to $f$. By the IH, the configuration $(f', m, p', p)$ must be generated at some stage. Now there are two possibilities: if the summary pair $\langle p, q\rangle$ exists at this stage, then FOLLOW-EDGE code ensures that $(f', n', p', q)$ is generated. If not, then $(f', m, p', p)$ is added to the call sites for $f$. When the summary pair is generated later for $f$, this configuration is put back on the worklist and the summary pair is available when it is re-processed. □

**Theorem 1.** *The algorithm reports an error at a configuration $(f, n, p, q)$, for some $p, q$, if and only if there is a program execution ending at a global state with form $x(f, n)$, (for some $x$) labeled with trace $t$, such that the automaton has a rejecting run on $t$.*

*Proof.* (left-to-right) Suppose that the algorithm reports an error at $(f, n, p, q)$. By Lemma 1, there is a program execution to a global state of the form $x(f, n)$ (for some $x$), and a run of the automaton on the associated trace that ends in automaton state $q$. Since the procedure reports an error, $q$ must be a rejecting automaton state, hence this is a rejecting run.

(right-to-left) Suppose that there is a program execution $\gamma$ ending at a global state of the form $x(f, n)$ (for some $x$), labeled with trace $t$ such that the automaton has a rejecting run on $t$. Let $q$ be the final state of the automaton run, and let $p$ be the state of the automaton corresponding to the last call to $f$ on $\gamma$. Then, by Lemma 3, the configuration $(f, n, p, q)$ must be generated by the procedure, and as $q$ is a reject state, the procedure will report an error at this configuration. □

**Theorem 2.** *The derivation graph resulting from the IncrFwd algorithm is exactly the graph generated by the Full analysis algorithm on the modified CFGs.*

*Proof.* (Sketch) Given the similarity between the algorithms, it is easy to show that any schedule of choices (and markings) from the workset in the checking algorithm corresponds with a schedule of choices (and generation) from the workset in the full analysis algorithm, and vice versa.

Precisely, one can show a bisimulation between the states of the Full and IncrFwd algorithm. In the bisimulation, algorithm states are related if (i) worksets are identical, (ii) for each function $f$, the summaries generated for $f$ in Full are identical to the marked summaries for IncrFwd, and (iii) states visited in Full are exactly those marked in IncrFwd. □

**Theorem 3.** *The derivation graph after the IncrBack algorithm is an over-approximation of the graph generated by the full analysis algorithm on the modified CFGs, but has precisely the same set of error configurations.*

*Proof.* The proof is based on reverse induction on the depth of SCC's in the (acyclic) SCC decomposition graph. It uses the following stronger induction hypothesis: at each stage, the repaired derivation graph and summary pairs for functions in SCCs below the current SCC are identical to those generated by applying the full analysis algorithm with the modified CFGs and the old entry configurations.

Consider an SCC with $N$ levels. The IH is true initially as there are no SCCs at level $N + 1$. Consider the processing of an SCC $C$ at level $k$. Functions in $C$ can only access other functions in SCC's at levels $k$ or higher. From the proof of Theorem 2, we claim that the checking process is equivalent to the generation process *from the given entry configurations* for functions in $C$. The generation and checking processes both make use of summaries for functions in SCC's below $C$, but these are accurate by the induction hypothesis. Hence, the summaries and derivation graphs produced for functions in $C$ are also accurate.

The $main$ function is at level $0$. Applying the induction hypothesis to this SCC shows the first part of the claim. Thus, there can only be more error configurations than those generated by the full algorithm, and the excess configurations can be traced back to old entry configurations that are now unreachable. This situation is detected by RETRACE. After this final step, the error configurations are exactly those determined by the full analysis. □

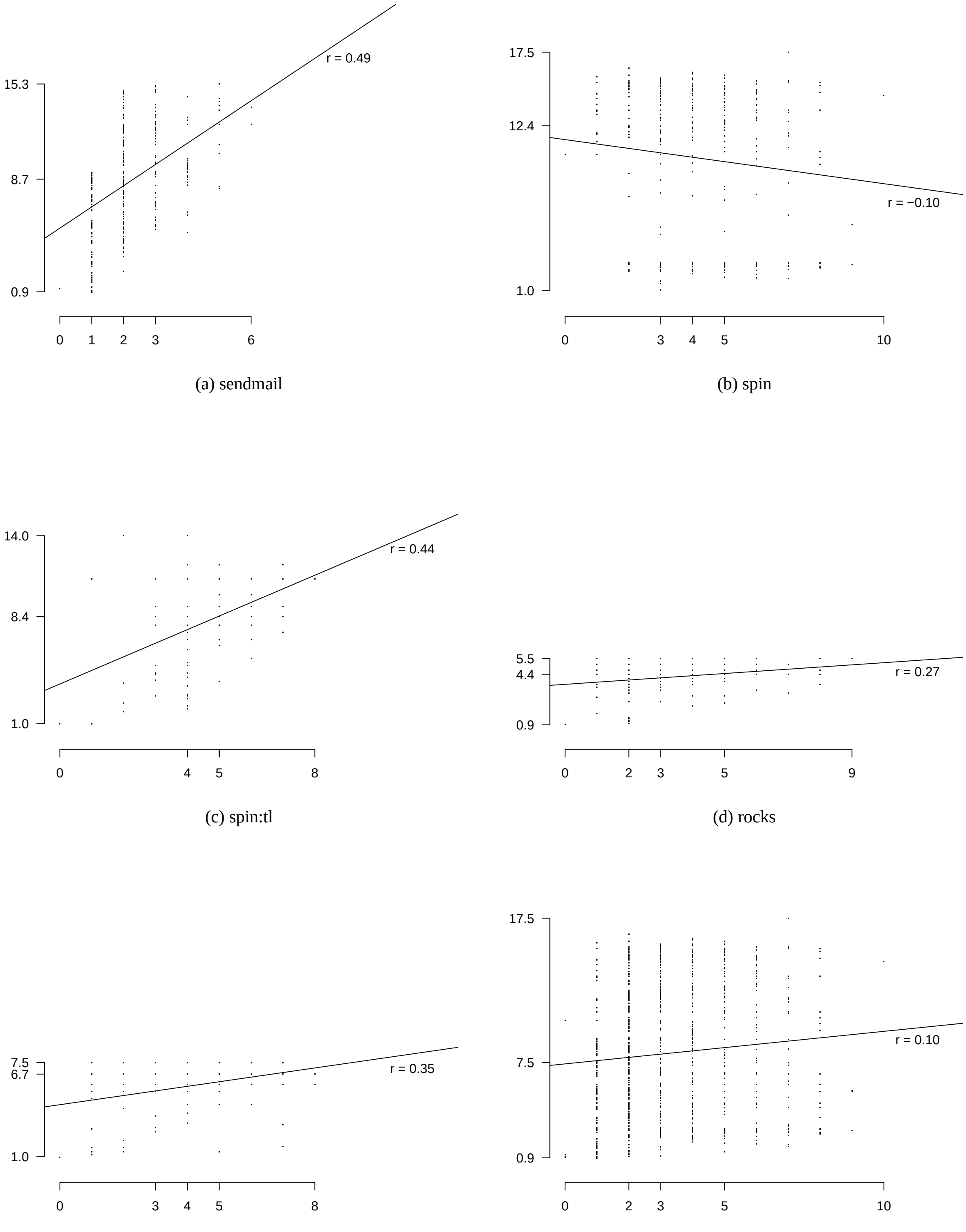
# B Experimental Data

Figures 7, 8, 9, 10, and 11 plot the experimental data in terms of, respectively, call depth (breadth-first distance from the entry function), relative number of CFG nodes modified, number of callers, number of callees, and total degree in the call graph (the sum of callers and callees). In each case the dependent variable (analysis speedup for IncrBack) is plotted on the $y$-axis and the independent variable is plotted on the $x$-axis. The $x$-axis is labelled with quantile values (i.e., the percentiles $0\%$ (minimum), $25\%$, $50\%$ (median), $75\%$, and $100\%$ (maximum)). The $y$-axis is labelled with minimum, median, and maximum speedup values.

Each plot includes a regression line and the correlation coefficient $r$. The quantity $r^2$ can be thought of as the probability that the observed variance is due to random variation. Thus, correlation coefficients close to $1$ and $-1$ represent strong linear relationships; correlation coefficients close to $0$ represent weak linear relationships.

In Fig. 7, every benchmark (except spin) shows a moderate correlation between the call depth of a function and the incremental speedup when the function is modified. In each case, the entry function (with call depth 0) shows no speedup at all. The way the incremental experiments were conducted, the initial analysis (with the entry function removed) misses every function in the program, so the incremental algorithm has no advantage over the Full algorithm. In real usage scenarios, where incremental changes would be less drastic, the speedup for an entry function should be less exceptional.

In Fig. 9, every benchmark (except spin) shows a fairly strong negative correlation between the log of the number of callers for a function and the incremental speedup. The correlation is also fairly strong (again, with the exception of spin) for the total degree of a function in the call graph (see Fig. 11). The correlation for the number of callees is weaker (see Fig. 10).



(a) sendmail

(b) spin

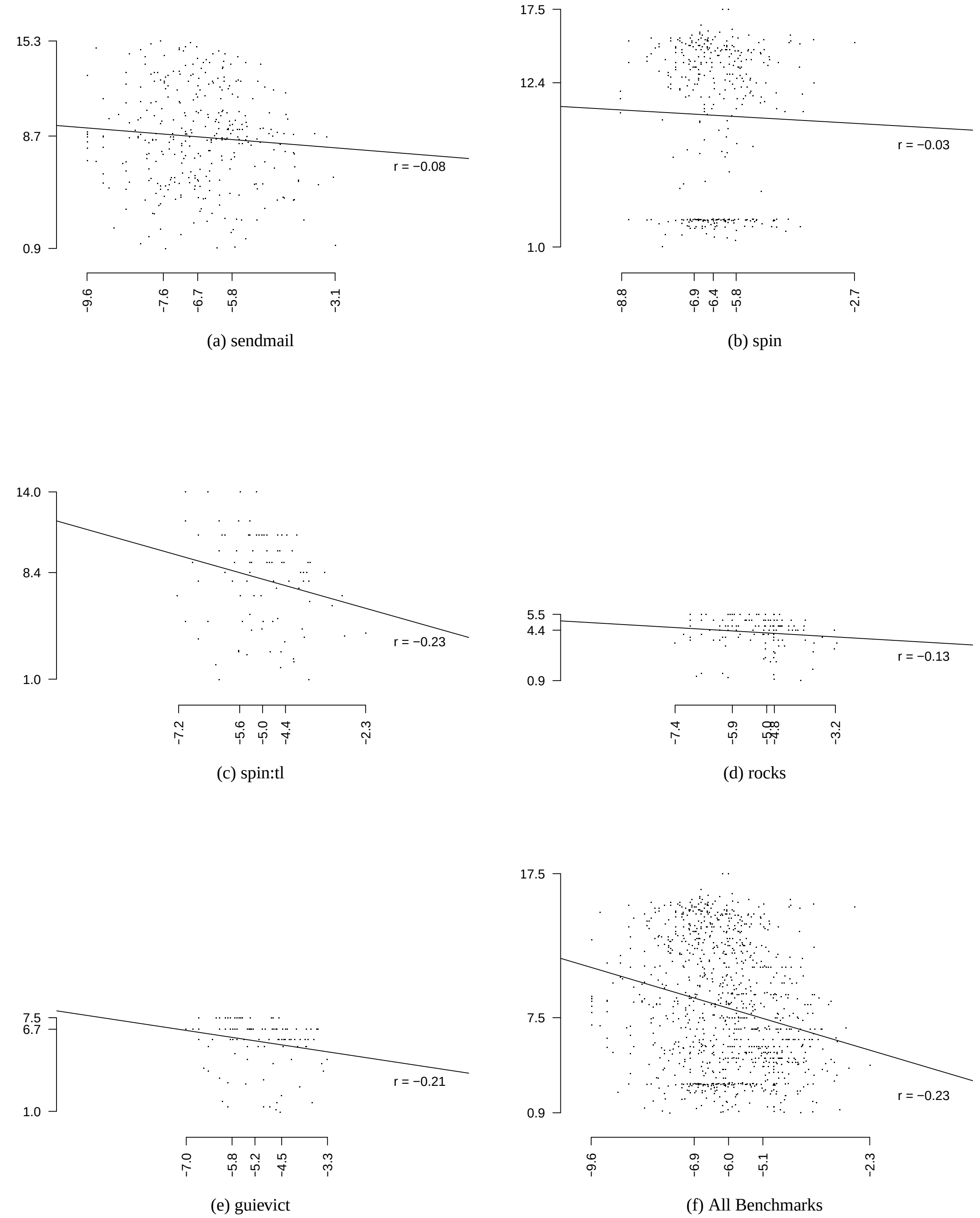
(c) spin:tl

(d) rocks

(e) guievict

(f) All Benchmarks

Figure 7: Function call depth vs. speedup for IncrBack. In (c), 5 is both the second (median) and third quantile.



(a) sendmail

(b) spin

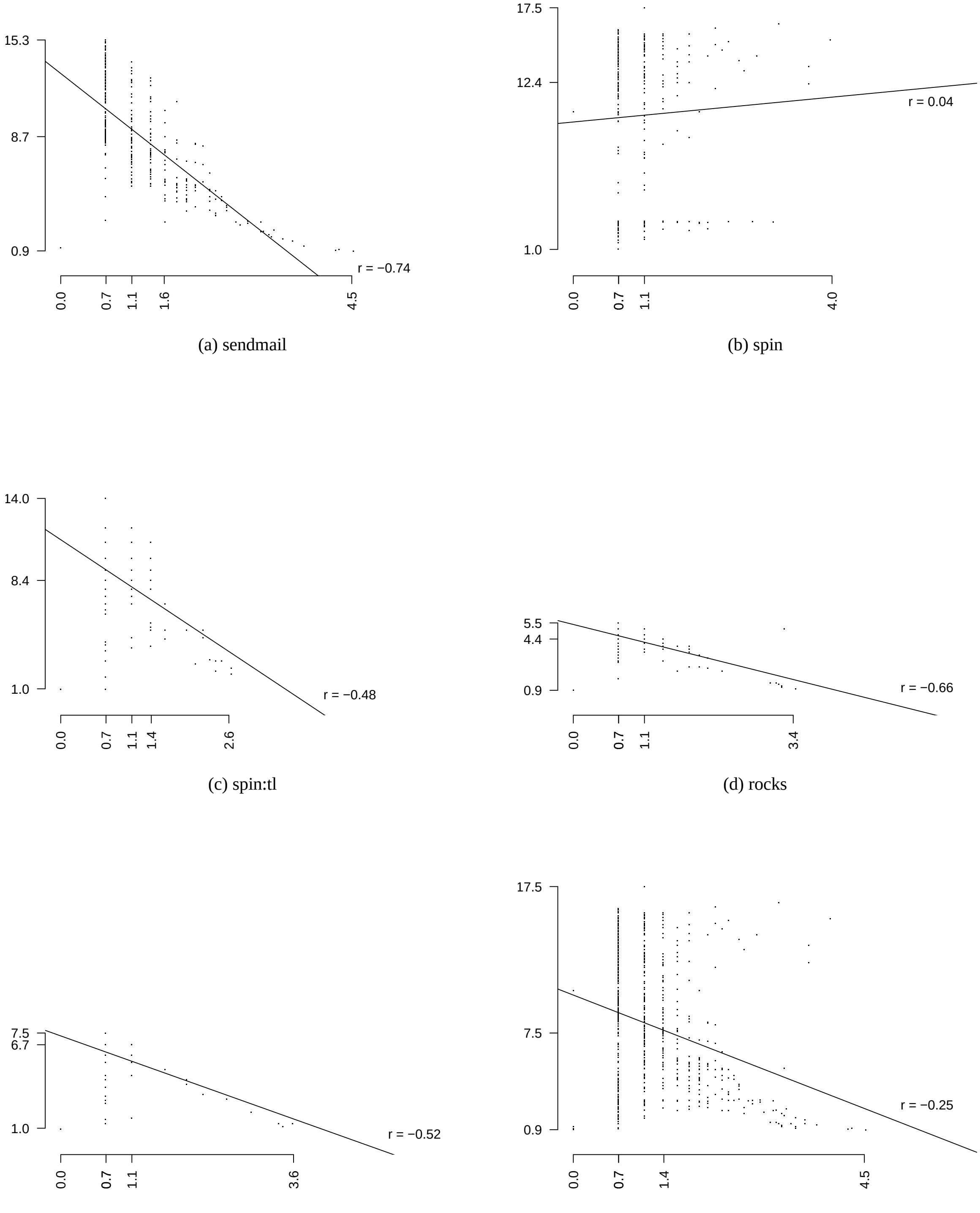
(c) spin:tl

(d) rocks

(e) guievict

(f) All Benchmarks

Figure 8: Proportion of CFG nodes in the modified function vs. speedup for IncrBack. The CFG data is plotted in log-scale: a point plotted at $x$ on the horizontal axis represents a incremental run in which a function with $Ne^x$ CFG nodes is modified (where $N$ is the total number of CFG nodes in the program).



(a) sendmail

(b) spin

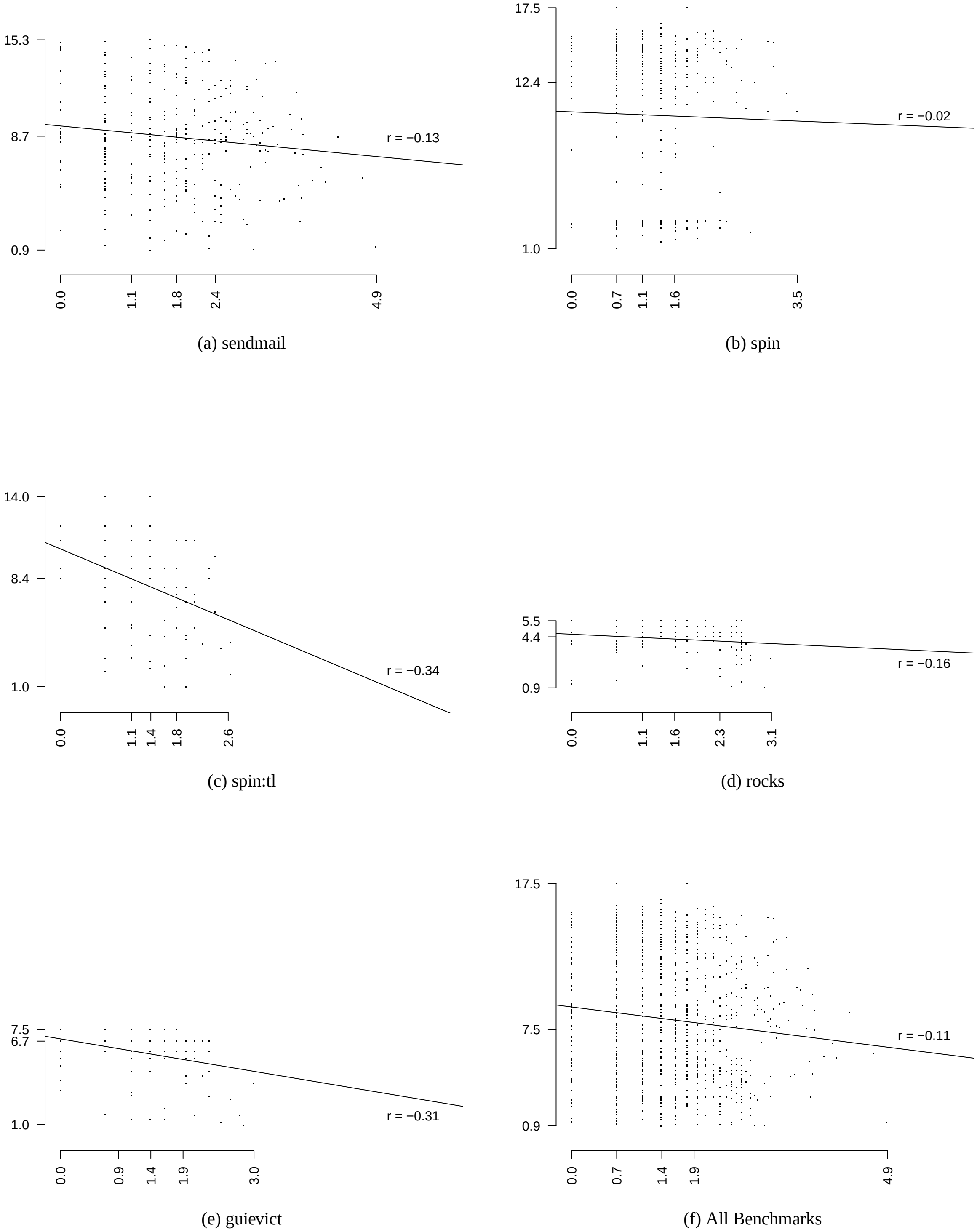
(c) spin:tl

(d) rocks

(e) guievict

(f) All Benchmarks

Figure 9: Number of callers vs. speedup for IncrBack. The horizontal axis is plotted in logscale and shifted to the right by 1; a point plotted at $n$ on the horizontal axis represents an incremental run on a function with $e^n - 1$ callers. In (b), (d), (e), and (f), $0.7$ is both the first and second (median) quantile, representing the large number of functions with a single caller ($e^{0.7} - 1 \approx 1$).



(a) sendmail

(b) spin

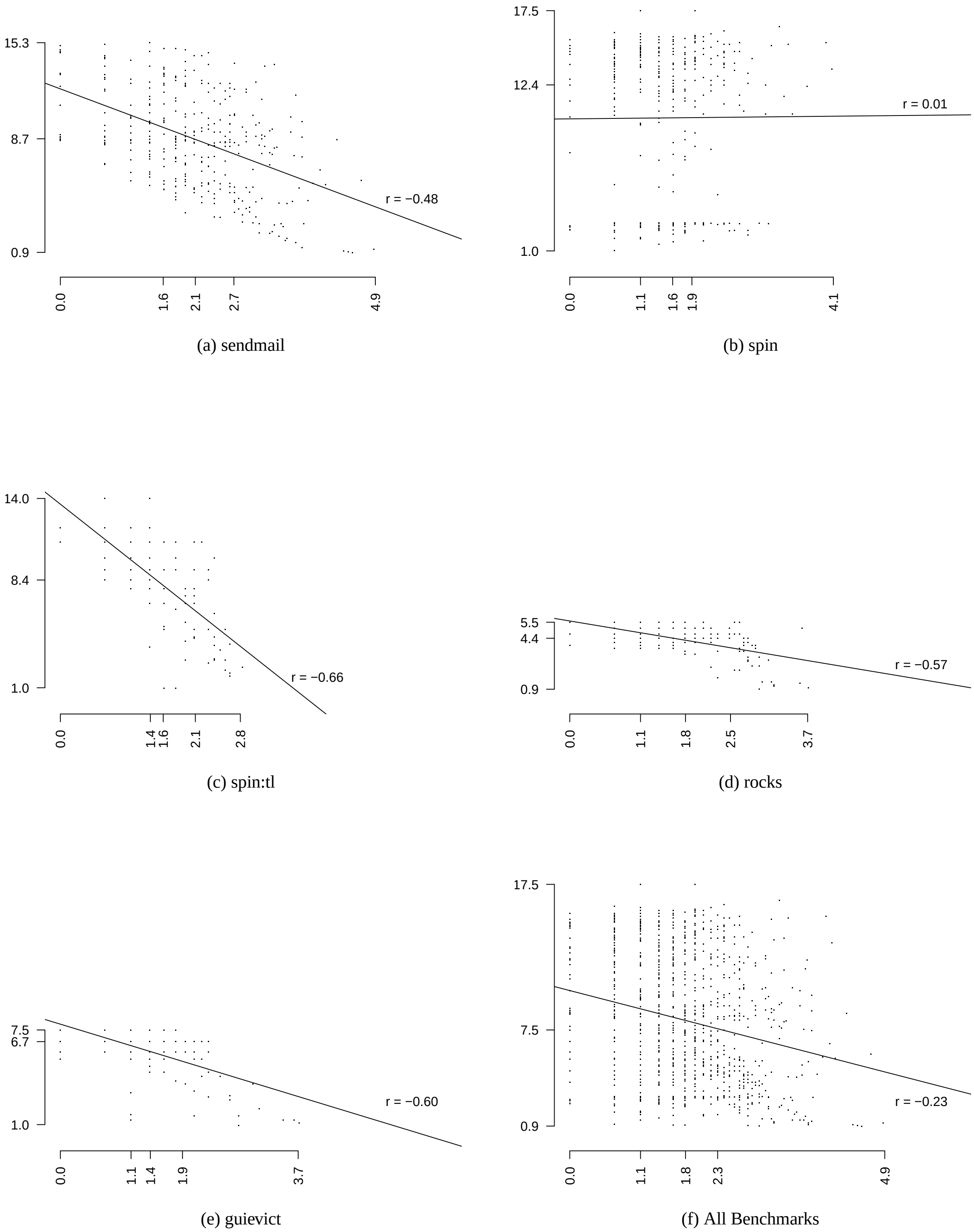
(c) spin:tl

(d) rocks

(e) guievict

(f) All Benchmarks

Figure 10: Number of callees vs. speedup for IncrBack. The horizontal axis is plotted in logscale and shifted to the right by 1; a point plotted at $n$ on the horizontal axis represents an incremental run on a function with $e^n - 1$ callees.



(a) sendmail

(b) spin

(c) spin:tl

(d) rocks

(e) guievict

(f) All Benchmarks

Figure 11: Total degree of a function in the call graph vs. speedup for IncrBack. The degree data is plotted in log-scale: a point plotted at $n$ on the horizontal axis represents an incremental run on a function with $e^n$ callers and callees.