Model Checking with Multi-Valued Logics

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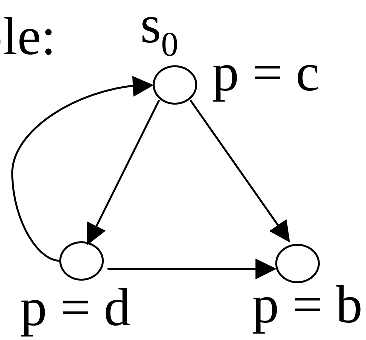
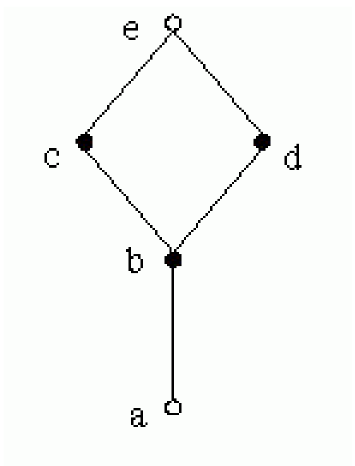
Multi-Valued Model Checking: Definition

- Kripke structure K where, in every state, every atomic proposition is mapped to an element of a lattice L.
  - Example:

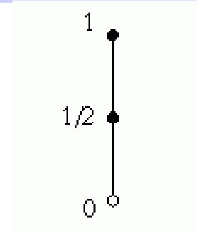




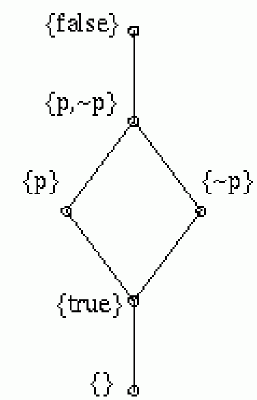
- Multi-valued temporal logic:
  - Syntax: unchanged
  - Semantics: unchanged except $\wedge$ is meet in L, $\vee$ is join in L
  - Example: $\phi = p \wedge EX\ p$
- $[K \models \phi]$ ("model checking") returns a value in L.
  - Example: $[(K,s_0) \models p \wedge EX\ p] = (c \wedge (d \vee b)) = b$

Motivation: Applications

- Model Checking using 3-valued abstractions
  - Automatically abstract a program into a 3-valued model K
  - Check any temporal property ϕ on this model
  - If [K ⊨ ϕ] = ½, refine the model and repeat the process

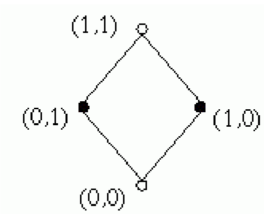


L_3



- Temporal-logic query checking
  - Given a “query” ϕ (ex: AG?), what is the set of strongest propositional formulas f (built from P) such that K ⊨ ϕ[? ← f]

- Multi-viewpoints model checking
  - What properties do different experts agree on?



$L_{2,2}$

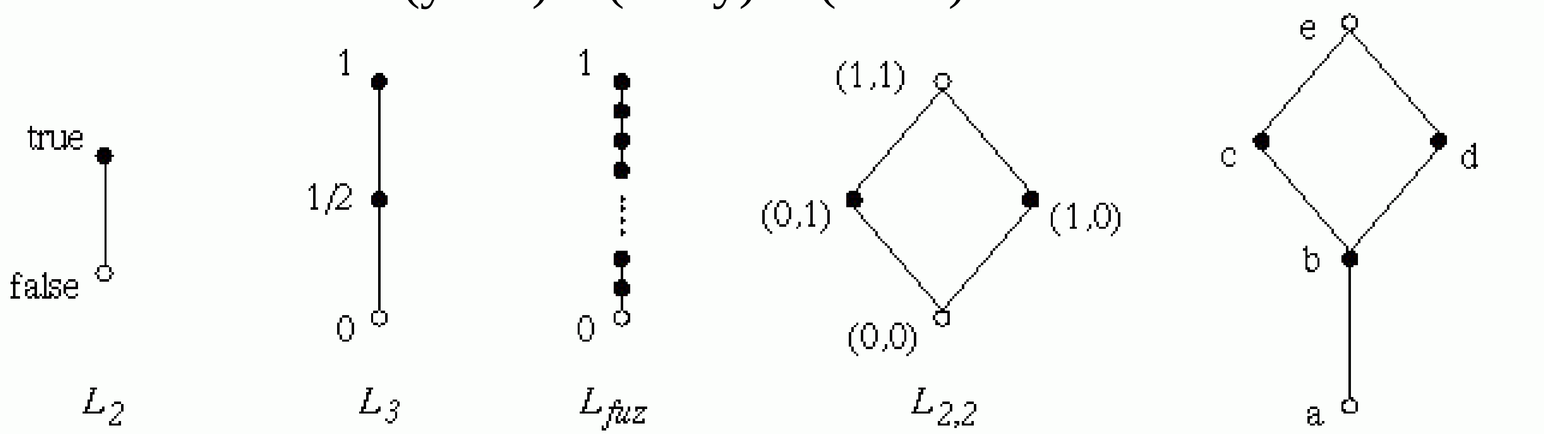
- All these problems reduce to “multi-valued model checking”

Two Approaches

- By reduction
  - Idea: reduction to several standard, 2-valued model-checking problems
  - Advantage: re-use of existing model checkers
  - New result: simple and general method for reduction
- Direct (automata-theoretic) approach
  - Idea: represent the formula by an EAA, and compute product with K
  - Advantage: works in a more “demand-driven” way
  - New result: maximum-value theorem for EAA and general automata-theoretic approach to multi-valued model checking

Lattices and Negation

- We consider finite (hence complete) distributive lattices.
  - Complete: $\forall$ X $\subseteq$ L: $\wedge$\{X\} and $\vee$\{X\} exist in L
  - Distributive: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$



- A *join-irreducible element* x of a distributive lattice L is an element that is not $\perp$ and for which $(x = y \vee z) \Rightarrow (x = y$ or $x = z)$

- DeMorgan lattice: every $x \in L$ has a unique complement $\neg x$ such that $\neg\neg x = x$, DeMorgan's laws hold, and $(x \leq y) \Rightarrow (\neg y \leq \neg x)$

Reduction Method (Approach 1)

- Given K, $\phi \in \mu$-calculus, and a finite distributive DeMorgan L:
  - Push $\neg$ inwards (using DeMorgan laws) to get ϕ in positive normal form.
  - $\forall x \in L$, define K_x as K except that $\theta_x(s)(p) = \theta(s)(p) \geq x$
  - Let J(L) denote the (finite) set of join-irreducible elements of L.
  - Lemma 1: Given K over L, s in K, $x \in J(L)$: $(K_x,s) \models \phi \Leftrightarrow x \leq [(K,s) \models \phi]$
  - Theorem 1: $[(K,s) \models \phi] = \vee\{x \in J(L) \mid (K_x,s) \models \phi\}$
  - Theorem 2: Given a TL, multi-valued model checking $[(K,s) \models \phi]$ for TL has the same complexity in K and ϕ as traditional model checking for TL, and can be done in time $O(|J(L)|)$.

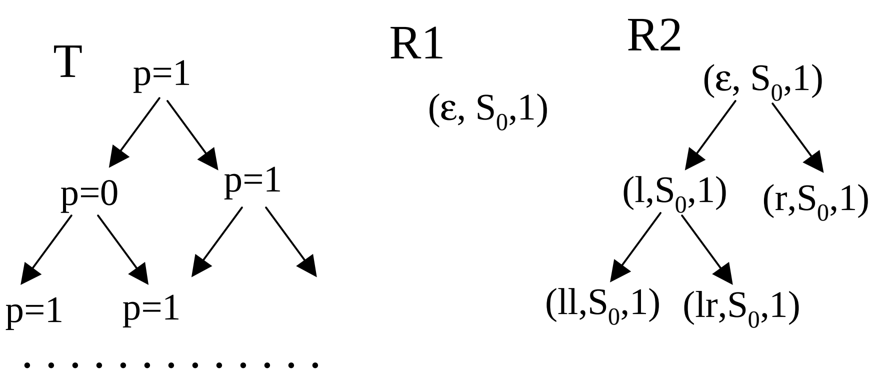
- Notes:
  - Sometimes complexity in $|J(L)|$ is better than linear…
  - These results can easily be extended to multi-valued transitions…

Comparison with Related Work

- Generalizes reduction methods for specific lattices
  - 3-valued model checking [BrunsGodefroid00]
  - Several other lattices [KonikowskaPenczek02]
- Simplifies other reduction method using join-irreducible elements
  - [GurfinkelChechik03]
- Extends work on “many-valued modal logics” [Fitting92]
  - Reduction to standard Kripke structure vs. “multi-expert models”
  - Join-irreducible elements instead of “proper-prime filters”
  - Fixpoint modal logic vs. modal logic
  - Different treatment of negation (DeMorgan lattices vs. Heyting algebras)
- Different from work on “AC-lattices” [HuthPradhan02]

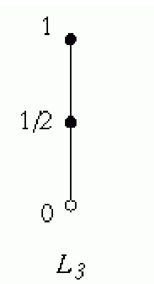
Extended Alternating Automata (Approach 2)

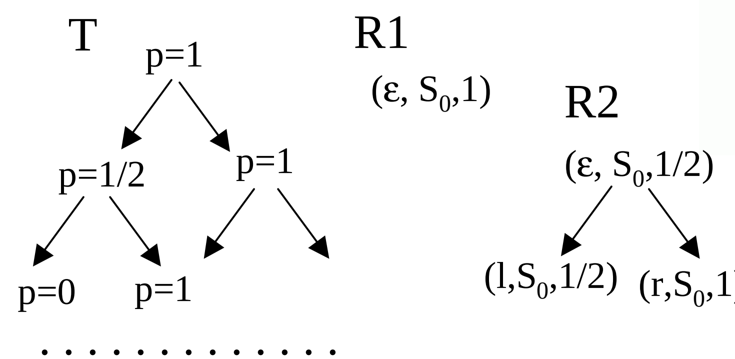
- Alternating Automaton $A=(\Sigma,S,s_0,\rho,F)$ with input alphabet Σ, transition function ρ, acceptance condition F
- Ex: $\rho(s_0,\sigma,2)= \sigma(p) \vee ((l,s_0)\wedge(r,s_0))$ and $F=\{\}$ (equivalent to AFp in CTL)
- Run: ∞ input tree T $\rightarrow$ run tree R



- T is accepted by A (denoted $T\in L(A)$) if A has an accepting run R on T:
  - every ∞ branch of R satisfies F.

- Extended Alternating Automaton [BrunsGodefroid01]: same as AA except ρ is defined on L with $\wedge$ and $\vee$
- Run: ∞ input tree T $\rightarrow$ run tree R labeled with non-$\perp$ elements of L



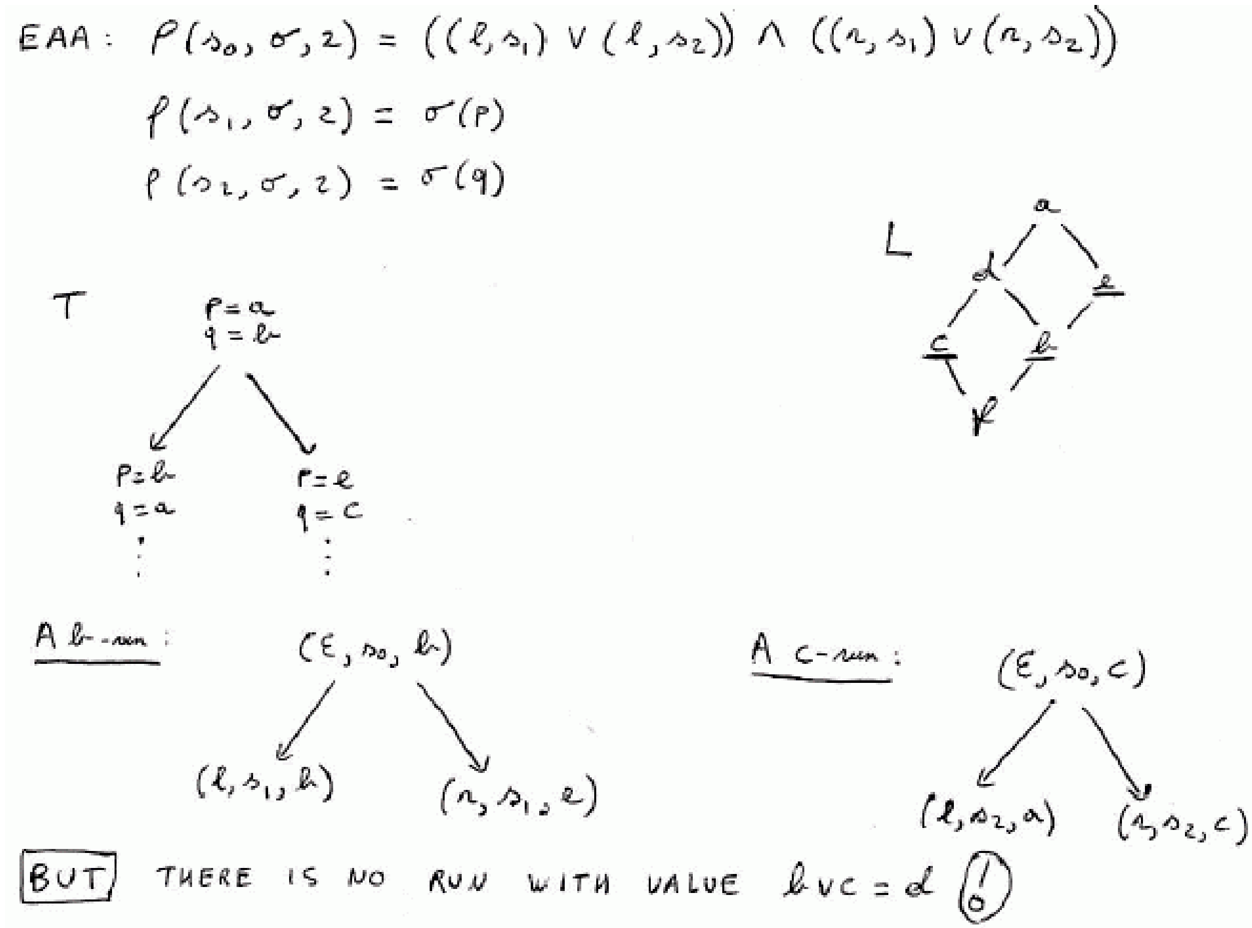


- T is accepted by A with value v (denoted $T \in L_v(A)$) if A has an accepting v-run R on T:
  - v labels the root node of R
  - every ∞ branch of R satisfies F.

Maximum-value Theorem

- Thm: Let A be a finite tree EAA over L, and let T be an infinite input tree. Then the subset $\{v \mid T \in L_v(A)\}$ of L has a maximum value Max(A,T).

- Note: nontrivial!



Proof Idea

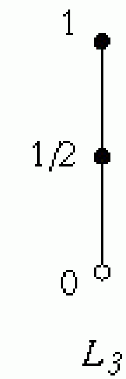
- Define a lattice V of valuation trees ordered by the sub-tree relation and $\leq$ on L. Since L is complete, V is a complete lattice.
- Define a function F: $V \rightarrow V$ that computes the transition function ρ
- Runs correspond to fixpoints of F.
- Apply Knaster-Tarski's theorem to F (order-preserving on V): "the join R of all runs (fixpoints of F) is a run (fixpoint of F)."
- Problem: R may not be accepting! (since the join of infinitely-many finite paths may not be accepting…)
- Solution: provide a construction to eliminate all infinite non-accepting paths in R while preserving the label of its root node…

Model Checking with EAA

- Automata-theoretic approach to multi-valued model checking (extends [KupfermanVardiWolper00]):
  - Translate ϕ into a tree EAA A_ϕ such that $[T \models \phi] = Max(A_\phi, T)$ (translation similar to the traditional one except for atomic propositions)
  - Compute the product $A_{K,\phi}$ of K and A_ϕ (a word EAA on 1-letter alphabet)
  - Theorem: $[K \models \phi] = Max(A_{K,\phi})$
  - Computing $Max(A_{K,\phi})$ has the same complexity in $|A_{K,\phi}|$ as checking language emptiness in regular word AA on 1-letter alphabet, and can be done in time $O(|h(L)|)$.
  - Example: if Buchi acceptance condition, quadratic time in $|A_{K,\phi}|$, or even linear time in $|A_{K,\phi}|$ if the EAA is also 'weak' (e.g., for CTL).

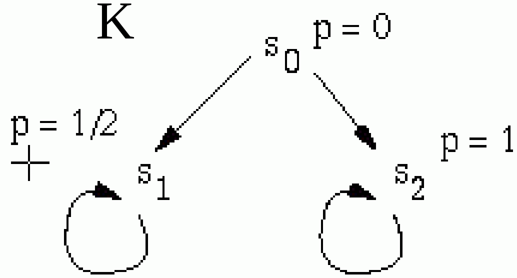
Example

- Consider the lattice L_3

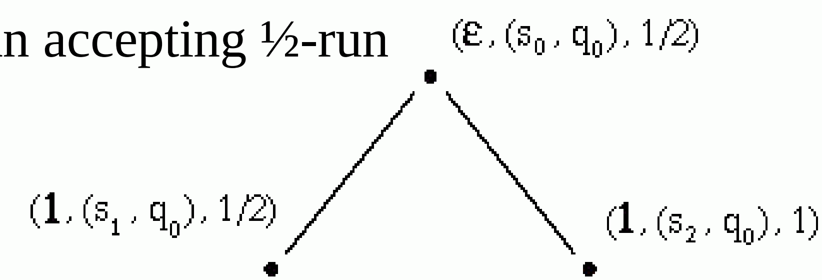


- Consider the formula ϕ = AF p (= $\mu X.p \vee \Box X$)
- A_ϕ is a tree EAA on L_3 with $\rho(q_0,\sigma,2) = \sigma(p) \vee ((l,q_0) \wedge (r,q_0))$ and F={}
- Given K below, $A_{K,\phi}$ is a word EAA on 1-letter alphabet {a} with $\rho((s_0,q_0),a,1) = 0 \vee ((s_1,q_0) \wedge (s_2,q_0))$, $\rho((s_1, q_0),a,1) = 1/2 \vee (s_1,q_0)$, $\rho((s_2,q_0),a,1) = 1 \vee (s_2,q_0)$, and F={}
- $[K \models \phi] = Max(A_{K,\phi}) = 1/2$

K



An accepting ½-run



Summary and Conclusions

- Summary: two approaches to multi-valued model checking
  - By reduction
    - Advantage: re-use of existing model-checking tools
    - New result: simple and general method based on join-irreducible elements for finite distributive DeMorgan lattices and full μ-calculus
  - Direct, automata-theoretic
    - Advantage: more “on-the-fly”/demand-driven
    - New result: maximum-value theorem for EAA and general automata-theoretic approach for DeMorgan lattices and full μ-calculus
- Future work:
  - Complementation of EAA…
  - Detailed study of algorithms for computing Max(EAA) (infinite games + lattice equations)…
  - Other applications: quantitative games for resource optimization?