Achievable Performance of Dynamic Channel Assignment Schemes under Varying Reuse Constraints

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***Abstract*—We introduce a reward paradigm to derive novel bounds for the performance of dynamic channel assignment (DCA) schemes. In the case of uniform reuse, our bounds closely approach the performance of maximum packing (MP), which is an idealized DCA scheme. This suggests not only that the bounds are extremely tight, but also that no DCA scheme, however sophisticated, will be able to achieve significant capacity gains beyond those obtained from MP.**

Our bounds extend to varying reuse scenarios which may arise in the case of reuse partitioning techniques, measurement-based DCA schemes, or micro-cellular environments. In these cases, the bounds slightly diverge from the performance of MP, which inflicts higher blocking on outer calls than inner calls, but not to the extent required to maximize carried traffic. This reflects the inherent tradeoff that arises in the case of varying reuse between efficiency and fairness. Asymptotic analysis confirms that schemes which minimize blocking intrinsically favor inner calls over outer calls, whereas schemes which do not discriminate among calls inevitably produce higher network-average blocking. Comparisons also indicate that DCA schemes are crucial in fully extracting the potential capacity gains from tighter reuse.

***Index Terms*—Achievable performance, call blocking, dynamic channel assignment, Erlang bound, Maximum Packing, performance bounds, reuse partitioning, revenue bound, varying reuse constraints, trunk reservation.**

I. INTRODUCTION

THE use of wireless services has been expanding at a tremendous rate. The dramatic growth is fueled not only by the proliferation of traditional voice users but also the introduction of new high-speed data services. The capacity expansion has not been keeping equal pace with the demand, creating a strong incentive to squeeze the most out of the existing network resources. With further growth anticipated, the drive for efficient resource utilization will certainly persist, since the available spectrum for wireless communications is quite limited, while the cost of new infrastructure is significant.

Numerous approaches to increase efficiency have been proposed, such as Dynamic Channel Assignment (DCA) schemes, reuse partitioning techniques, measurement-based algorithms, and micro-cellular networks. Simulation results indicate that these approaches may achieve substantial capacity gains. To resolve basic design issues, however, it is crucial to gain understanding at a more fundamental level of the most efficient ways of resource utilization. In the present paper, we derive novel performance bounds which provide insight into the potential capacity gains from a more fundamental perspective.

The model that we adopt is that of the circuit-switched networks currently deployed for carrying voice traffic. Suitably modified, most of the insights carry over to the packet-oriented systems that have been proposed for supporting high-speed data users. In these systems, backlogged packets are queued, in contrast to calls that are lost, while resource management operates on a faster time scale to be able to adequately respond to the bursty nature of data traffic. We refer to a companion paper [1] which explores these issues in greater detail.

A. DCA Schemes

One approach to enhance capacity in wireless networks, is to allow channels to be assigned in a more flexible manner. Most existing networks operate according to Fixed Channel Assignment (FCA) schemes [10]. In FCA, channels are statically allocated to cells, subject to certain reuse constraints. The reuse constraints determine which pairs of cells may use the same channel simultaneously, based on interference considerations.

In DCA, in contrast, channels are not permanently allocated to cells, but may be dynamically diverted to respond to fluctuations in the offered traffic [10]. Besides the potential capacity improvements, the flexibility of DCA schemes greatly reduces the need for frequency planning. Detailed frequency planning is seriously hampered by the fact that in practice it may be extremely difficult to estimate the offered traffic and to predict the interference conditions. This is in particular true in micro-cellular environments. Unreliable information may necessitate a conservative approach, causing a reduction in capacity. In the present paper, however, we restrict the attention to the potential capacity improvements when the offered traffic is known in advance and does not have any spatial or temporal variations.

Maximum packing (MP) is an idealized DCA scheme which was introduced by Everitt and MacFadyen [2]. MP accepts calls whenever possible, even if this involves rearranging the channels assigned to calls in progress. Kelly [7] presents an exact analysis of MP on a doubly infinite strip, in which two adjacent cells cannot simultaneously use the same channel. The results show that even for uniform offered traffic, MP outperforms FCA, unless the load exceeds a certain critical value. Jordan and Khan [5] and Kind *et al.* [9] report a similar observation, which has led to the belief that there might actually be hybrid schemes that outperform MP. Our results however indicate that *no* DCA

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scheme will be able to achieve significant capacity gains beyond those obtained from MP.

B. Tighter Reuse

Another approach to improve efficiency, is to allow tighter reuse of channels. The conventional cell-by-cell reuse constraints are based on the interference levels which mobiles would experience under worst-case conditions. Tighter reuse may be achieved by taking the actual mobile locations into account. In reuse partitioning for example, cells are split into inner and outer regions. The smaller radius of the inner regions allows for lower powers and thus tighter reuse of channels. Measurement-based algorithms may be viewed as a limiting form of reuse partitioning. Tighter reuse of channels is also a primary source contributing to the capacity gains in micro-cellular networks. The model that we adopt in the present paper is that of cells split into inner and outer regions. Most of the observations however pertain to any of the variants mentioned above.

Our results show that MP-type strategies fail to fully extract the potential capacity gains in these scenarios. MP inflicts higher blocking on outer calls than inner calls, but not to the extent required to maximize carried traffic, see also Shimada *et al.* [14] and Valenzuela [15]. The first of these two papers proposes various mechanisms to alleviate the spatial imbalance in blocking, at the expense of higher network-average blocking. This reflects the inherent tradeoff that arises in the case of varying reuse between efficiency and fairness. Asymptotic analysis confirms that schemes which minimize blocking intrinsically favor inner calls over outer calls, whereas schemes which do not discriminate among calls inevitably produce higher network-average blocking.

Nothing prevents the tighter reuse of channels to be integrated with the use of DCA schemes. In fact, a key observation from our paper is that the use of DCA schemes is crucial in fully extracting the potential capacity gains from tighter reuse. We refer to Katzela and Naghshineh [6] for a comprehensive survey of DCA schemes and reuse partitioning techniques. It is finally worth mentioning that besides the potential capacity gains there are other important issues in evaluating the merits of DCA schemes and reuse partitioning techniques, such as additional complexity and hand-offs.

C. Bounds

As a rule, exact analysis of DCA schemes is prohibitively demanding. In fact, to the best of our knowledge, MP on a doubly-infinite strip is one of the very few exceptions. The prohibitive complexity of exact analysis motivates the construction of performance bounds as an alternative way of gaining insight into the potential capacity gains from DCA schemes.

An example is the Erlang bound, which was first derived in Whiting [16], and later studied in Raymond [13]. The Erlang bound provides a lower limit on the network-average blocking under any DCA scheme, which may be obtained as the solution to a certain linear program. Frodigh [3] derives bounds for measurement-based DCA schemes in linear networks. The bounds are based on a ‘snapshot’ analysis, determining the maximum number of calls a particular scheme could accommodate of those offered to the system, including calls that may have been blocked or dropped at some earlier stage. Xu and Akansu [18] and Zander and Eriksson [19] obtain asymptotic lower and upper bounds for measurement-based DCA schemes in planar networks. The bounds are derived from geometrical arguments, treating traffic as a deterministic, infinitely-divisible fluid. In the present paper, we obtain a novel family of bounds which fully capture the dynamics and the stochastic nature of the system.

In summary, the paper is organized as follows. In Section II, we present a more detailed model description, and briefly review the derivation of the Erlang bound. We also provide some basic examples illustrating how the Erlang bound may be calculated. Subsequently, we examine the achievable carried traffic region to understand why the Erlang bound may not always be tight. In Section III, we introduce a reward paradigm which paves the way for the construction of sharper bounds. We revisit the examples studied in Section II to illustrate how the revenue-based bounds may be used to improve upon the Erlang bound. Section IV specializes the results to symmetric, possibly infinite networks. We present numerical results for scenarios with uniform and varying reuse in Sections V and VI, respectively. In Section VII, we investigate the tradeoff between efficiency and fairness that arises in the case of varying reuse. Finally, in Section VIII, we summarize the main conclusions.

II. The Erlang Bound

We first present a more detailed model description. We consider a cellular network of arbitrary topology. The cells, which are indexed by the set $\mathcal{I}$, share a pool of C channels. Users in cell i generate calls as a Poisson process of rate ν_i. All calls have exponentially distributed holding times with unit mean.

When a user generates a call, the admission policy determines whether to accept or reject it. If accepted, the call is carried for the complete duration of the holding time. In case a call is rejected, the user does not make any retrials.

We assume that the admissible states of the network satisfy the constraints $\sum_{i\in\mathcal{C}} n_i \leq C$ for all $\mathcal{C} \in \Omega$, with n_i denoting the number of calls in cell i. The set Ω is the collection of *cliques*, which are defined as the subsets $\mathcal{C}$ of $\mathcal{I}$ such that no two users within $\mathcal{C}$ can share a channel.

Denote by

$$\mathrm{Erl}(\nu; C) = \frac{\nu^C}{C!} \Bigg/ \sum_{j=0}^{C} \frac{\nu^j}{j!} \tag{1}$$

the Erlang blocking formula for offered traffic ν and C channels. Notice that $\mathrm{Erl}(\nu; C)$ is the blocking in FCA for offered traffic ν and C channels per cell.

As shown in Whiting [16], the Erlang bound provides a lower limit on the network-average blocking under any admission scheme. It may be obtained as the solution to the following linear program:

$$\min \bar{B} \equiv \sum_{i\in\mathcal{I}} \nu_i B_i \Bigg/ \sum_{i\in\mathcal{I}} \nu_i \tag{2}$$

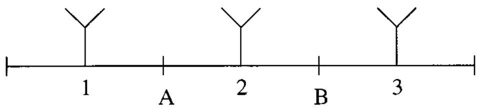

Fig. 1. Three-cell linear network.

$$\text{sub} \sum_{i \in \mathcal{C}} \nu_i B_i \geq \sum_{i \in \mathcal{C}} \nu_i \operatorname{Erl}\left(\sum_{i \in \mathcal{C}} \nu_i; C\right) \text{ for all } \mathcal{C} \in \Omega \quad (3)$$

$$0 \leq B_i \leq 1 \quad \text{for all } i \in \mathcal{I} \quad (4)$$

with the variables B_i representing the probability of call blocking in cell i under some arbitrary admission policy.

The key constraints are provided by the inequalities (3), which are obtained by considering each clique $\mathcal{C} \in \Omega$ in isolation. Since no two users within a clique can share a channel, we cannot accommodate more than C calls in any one clique simultaneously. Thus, we can never reject fewer calls in a clique $\mathcal{C} \in \Omega$ than the number of blocked calls for a single group of C channels offered traffic $\sum_{i \in \mathcal{C}} \nu_i$. This number is determined by the Erlang-B formula (1).

In fact, the Erlang bound would still apply if we wished to consider the sum of blocked and *dropped calls*. Even if call dropping were permitted, we can never lose fewer calls in total in a clique than the number of blocked calls for a single group of C channels.

We now provide some basic examples illustrating how the Erlang bound may be calculated.

Example 2.1: Three-Cell Linear Network: Consider the three-cell network depicted in Fig. 1. A channel cannot be used simultaneously in two adjacent cells, i.e., the cliques are $A = \{1, 2\}$ and $B = \{2, 3\}$. Thus, the clique constraints are $\nu_1 B_1 + \nu_2 B_2 \geq (\nu_1 + \nu_2) \operatorname{Erl}(\nu_1 + \nu_2; C)$ and $\nu_2 B_2 + \nu_3 B_3 \geq (\nu_2 + \nu_3) \operatorname{Erl}(\nu_2 + \nu_3; C)$. An optimal solution to the linear program is $B_1 = B_3 = 0$, $B_2 = (\nu_2 + \nu_{\max}/\nu_2) \operatorname{Erl}(\nu_2 + \nu_{\max}; C)$, with $\nu_{\max} := \max\{\nu_1, \nu_3\}$. This yields the bound

$$\bar{B} = \frac{\nu_2 + \nu_{\max}}{\nu_1 + \nu_2 + \nu_3} \operatorname{Erl}(\nu_2 + \nu_{\max}; C).$$

For uniform offered traffic ν, the bound reduces to $\bar{B} = (2/3) \operatorname{Erl}(2\nu; C)$. For $C = 10, \nu = 5$ for example, we obtain $\bar{B} \approx 0.143$.

The bound may be sharpened by adding the single-cell clique constraints $B_i \geq \operatorname{Erl}(\nu_i; C), i = 1, 2, 3$. An optimal solution to the linear program is then $B_1 = \operatorname{Erl}(\nu_1; C), B_3 = \operatorname{Erl}(\nu_3; C), B_2 = (1/\nu_2)[(\nu_2 + \nu_{\max}) \operatorname{Erl}(\nu_2 + \nu_{\max}; C) - \nu_{\max} \operatorname{Erl}(\nu_{\max}; C)]$. (The latter fact follows from the convexity of the blocked traffic $\nu \operatorname{Erl}(\nu; C)$ as a function of the offered traffic ν, see Harel [4], which implies that $\max\{(\nu_1 + \nu_2) \operatorname{Erl}(\nu_1 + \nu_2; C) - \nu_1 \operatorname{Erl}(\nu_1; C), (\nu_2 + \nu_3) \operatorname{Erl}(\nu_2 + \nu_3; C) - \nu_3 \operatorname{Erl}(\nu_1; C)\} = (\nu_2 + \nu_{\max}) \operatorname{Erl}(\nu_2 + \nu_{\max}; C) - \nu_{\max} \operatorname{Erl}(\nu_{\max}; C)$.) This tightens the bound to

$$\bar{B} = \frac{1}{\nu_1 + \nu_2 + \nu_3} [(\nu_2 + \nu_{\max}) \operatorname{Erl}(\nu_2 + \nu_{\max}; C) + \nu_{\min} \operatorname{Erl}(\nu_{\min}; C)]$$

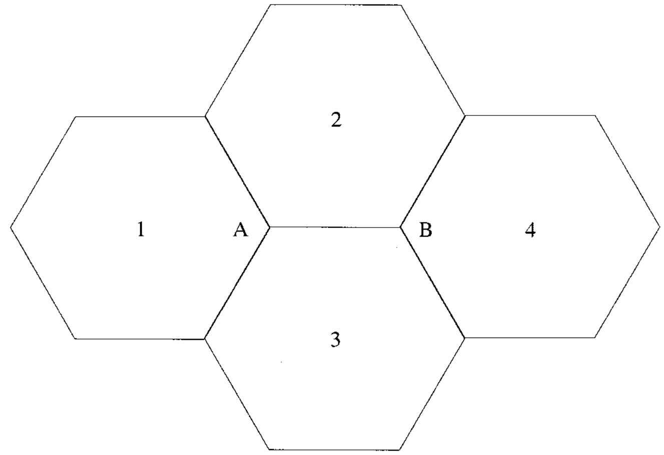

Fig. 2. Four-cell planar network.

with $\nu_{\min} := \min\{\nu_1, \nu_3\}$. For uniform offered traffic, the bound reduces to $\bar{B} = (1/3)(2 \operatorname{Erl}(2\nu; C) + \operatorname{Erl}(\nu; C))$. For $C = 10, \nu = 5$, we obtain $\bar{B} \approx 0.149$. Using Markov decision theory, we find that the minimum achievable blocking in fact is $\bar{B} \approx 0.215$. □

Example 2.2: Four-Cell Planar Network: Consider the four-cell network depicted in Fig. 2. A channel cannot be used simultaneously in two adjacent cells, i.e., the cliques are $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Now observe that the two center cells may be lumped together so that the network reduces to that of Example 2.1. Thus,

$$\bar{B} = \frac{\nu_2 + \nu_3 + \nu_{\max}}{\nu_1 + \nu_2 + \nu_3 + \nu_4} \operatorname{Erl}(\nu_2 + \nu_3 + \nu_{\max}; C)$$

with $\nu_{\max} := \max\{\nu_1, \nu_4\}$. For uniform offered traffic ν, the bound reduces to $\bar{B} = (3/4) \operatorname{Erl}(3\nu; C)$. For $C = 15, \nu = 5$ for example, we obtain $\bar{B} \approx 0.135\,24$.

By adding the single-cell clique constraints $B_i \geq \operatorname{Erl}(\nu_i; C), i = 1, 2, 3, 4$, the bound may be slightly tightened to

$$\bar{B} = \frac{1}{\nu_1 + \nu_2 + \nu_3 + \nu_4} [(\nu_2 + \nu_3 + \nu_{\max}) \times \operatorname{Erl}(\nu_2 + \nu_3 + \nu_{\max}; C) + \nu_{\min} \operatorname{Erl}(\nu_{\min}; C)]$$

with $\nu_{\min} := \min\{\nu_1, \nu_4\}$. For uniform offered traffic, the bound reduces to $\bar{B} = (1/4)(3 \operatorname{Erl}(3\nu; C) + \operatorname{Erl}(\nu; C))$. For $C = 15, \nu = 5$, we obtain $\bar{B} \approx 0.135\,28$. Using Markov decision theory, we find that the minimum achievable blocking is in fact $\bar{B} \approx 0.185\,79$.

A. Discussion

The Erlang bound as exemplified above may not always be tight. To understand why, we now examine the region of achievable carried traffic combinations. The clique constraints (3) underlying the Erlang bound may be rewritten

$$\sum_{i \in \mathcal{C}} \lambda_i \leq \sum_{i \in \mathcal{C}} \nu_i \left(1 - \operatorname{Erl}\left(\sum_{i \in \mathcal{C}} \nu_i; C\right)\right) \text{ for all } \mathcal{C} \in \Omega \quad (5)$$

with ν_i denoting the offered traffic in cell i and the variables $\lambda_i = \nu_i(1 - B_i)$ representing the carried traffic in cell i under

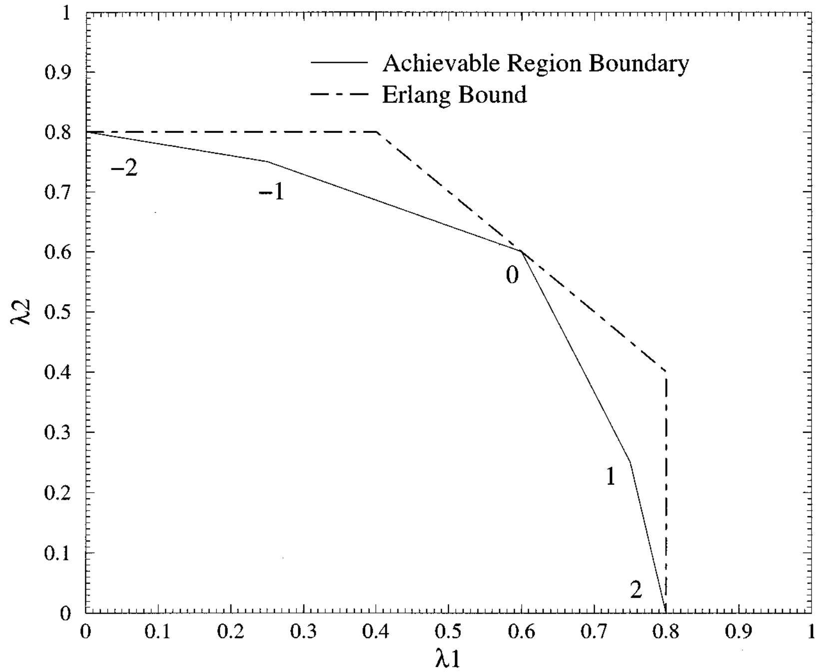

Fig. 3. Achievable carried traffic region for a single group of $C = 2$ channels offered two streams of traffic of rate $\nu = 1$ each.

some arbitrary admission policy. Now let us return to Example 2.1. The *outer* region in Fig. 3 delineates the set of all carried traffic pairs (λ_1, λ_2) that satisfy the constraints (5) for clique $A = \{1, 2\}$ for offered traffic $(\nu_1, \nu_2) = (1, 1)$. The diagonal boundary segment represents the constraint (5) corresponding to the clique $\mathcal{C} = \{1, 2\}$, i.e., $\lambda_1 + \lambda_2 \leq 1.2$, noting that $\mathrm{Erl}(2; 2) = 0.4$. The vertical boundary segment is determined by the constraint (5) for the single-cell clique $\mathcal{C} = \{1\}$, i.e., $\lambda_1 \leq 0.8$, noting that $\mathrm{Erl}(1; 2) = 0.2$. Similarly, the horizontal boundary segment corresponds to the constraint (5) for the single-cell clique $\mathcal{C} = \{2\}$. However, the *true* achievable carried traffic pairs for clique A, are demarcated by the *inner* region in the figure. This is the case if calls may be *blocked but not dropped*.

The piece-wise linear boundary of the *true* achievable region may be interpreted as follows. Consider a reward vector (w_1, w_2), with w_i representing the reward generated by each stream-i call that is carried. The reward-maximizing policy is then a *trunk reservation* strategy, see Lippman [11] and Miller [12]. Under trunk reservation, the calls of the lower-earning stream are rejected when there are no more than r free channels. This is the case for all nonnegative values of the reward vector w.

The carried traffic pairs for the class of trunk reservation strategies (there are five of them in this case) are represented by the vertices of the *inner* region in Fig. 3. They are labeled with the value of the corresponding trunk reservation parameter, taken negative when used against stream-1 calls. No carried traffic pair outside the inner region is achievable, since otherwise the optimality of the class of trunk reservation strategies would be contradicted. (Any pair within the inner region is in fact achievable through some probabilistic strategy, but this fact is not directly relevant for our purposes.)

The Erlang bound in Example 2.1 followed from the solution $(B_1, B_2) = (0.2, 0.6)$ to the linear program. Fig. 3, however, shows that the corresponding carried traffic pair $(\lambda_1, \lambda_2) = (0.8, 0.4)$ is infeasible. Thus, the Erlang bound may be strengthened if we replace the clique constraints (3) by the linear inequalities describing the boundary segments of the achievable region. This insight will be formalized in the next section.

Note that a different picture would emerge if call dropping were permitted. If pre-emption were allowed, then the achievable carried traffic pairs are exactly the vertices of the outer region in Fig. 3. Thus, the Erlang bound may not be tight because it fails to exclude carried traffic combinations which are only feasible if call dropping were permitted. Allowing for pre-emption, however, appears inappropriate as call dropping should be negligibly small for any sensible admission control scheme.

III. The Reward Bound

We now proceed with a formal statement of the proposed bounds. As we have seen in the previous discussion, we may use a reward paradigm as an insightful way of characterizing the achievable carried traffic region, and thus sharpening the Erlang bound. Specifically, suppose that each call carried in cell i generates a reward w_i. For any vector $w \in \mathcal{R}_+^{\mathcal{I}}$, denote by $R(w)$ the maximum achievable mean reward rate. Clearly, no admission policy can produce a higher mean reward rate than $R(w)$. This observation constitutes the basis for the next theorem.

Theorem 3.1: For any set $\mathcal{W} \subseteq \mathcal{R}_+^{\mathcal{I}}$, the carried traffic under any admission policy is bounded above by the optimum value $\lambda_{\mathcal{W}}$ of the following linear program

$$\max \sum_{i \in \mathcal{I}} x_i \tag{6}$$

$$\text{sub} \sum_{i \in \mathcal{I}} w_i x_i \leq R(w) \quad \text{for all } w \in \mathcal{W} \tag{7}$$

$$x_i \geq 0 \quad \text{for all } i \in \mathcal{I}. \tag{8}$$

Proof: The proof follows by interpreting the variables x_i as the carried traffic in cell i under some arbitrary admission policy. The objective function (6) then exactly represents the carried traffic. Constraint (7) is satisfied since the policy cannot produce a greater revenue than the maximum achievable reward rate. Hence, the optimum value of the linear program provides an upper bound for the carried traffic under any admission policy. □

Corollary 3.2: For any set $\mathcal{W} \subseteq \mathcal{R}_+^{\mathcal{I}}$, the carried traffic under any admission policy is bounded above by the optimum value $\mu_{\mathcal{W}}$ of the following linear program

$$\min \sum_{w \in \mathcal{W}} y(w) R(w) \tag{9}$$

$$\text{sub} \sum_{w \in \mathcal{W}} y(w) w_i \geq 1 \quad \text{for all } i \in \mathcal{I} \tag{10}$$

$$y(w) \geq 0 \quad \text{for all } w \in \mathcal{W}. \tag{11}$$

Proof: The proof follows by observing that (9)–(11) is the dual problem to (6)–(8). Strong duality then implies that $\lambda_{\mathcal{W}} = \mu_{\mathcal{W}}$. □

The main difficulty in evaluating the above bounds does usually not arise from solving the linear programs, but from computing the $R(w)$'s for a suitable set $\mathcal{W}$. Typically, determining $R(w)$ requires numerically solving a Markov decision problem with a state space in as many dimensions as the reward vector w has nonzero components. In certain cases, however, $R(w)$ may be obtained in closed form. For any clique $\mathcal{C} \in \Omega$ for example, $R(\chi^{\mathcal{C}}) = \sum_{i \in \mathcal{C}} \nu_i (1 - \text{Erl}(\sum_{i \in \mathcal{C}} \nu_i; C))$, with $\chi^{\mathcal{C}}$ denoting the characteristic vector of $\mathcal{C}$. From $\lambda_i = \nu_i(1 - B_i)$, we then also immediately see that the inequalities $\sum_{i \in \mathcal{I}} \chi_i^{\mathcal{C}} \lambda_i \leq R(\chi^{\mathcal{C}})$ are equivalent to the clique constraints $\sum_{i \in \mathcal{C}} \nu_i B_i \geq \sum_{i \in \mathcal{C}} \nu_i \,\text{Erl}(\sum_{i \in \mathcal{C}} \nu_i; C)$ in (3). Thus, for the set $\mathcal{W} := \bigcup_{\mathcal{C} \in \Omega} \{\chi^{\mathcal{C}}\}$, the above bounds coincide with the Erlang bound.

At the opposite side of the spectrum, $R(1, \ldots, 1)$ *equals* the maximum achievable carried traffic, but it is exactly the formidable complexity of calculating this quantity directly which motivated us to consider bounds. This contrast is characteristic of the tradeoff between the computational complexity of determining the $R(w)$'s and the tightness of the corresponding bounds.

For any subset $\mathcal{J} \subseteq \mathcal{I}$, denote $\mathcal{R}_+^{\mathcal{J}} := \{w \in \mathcal{R}_+^{\mathcal{I}} : w_i = 0$ for all $i \notin \mathcal{J}\}$. Now suppose that Π is the collection of subsets $\mathcal{D} \subseteq \mathcal{I}$ such that $R(w)$ can be obtained if $w \in \mathcal{R}_+^{\mathcal{D}}$. Define $\mathcal{W}^{\Pi} := \bigcup_{\mathcal{D} \in \Pi} \mathcal{R}_+^{\mathcal{D}}$ as the set of all reward vectors w for which $R(w)$ can be obtained. In case $\Pi \subseteq \Omega$, the collection of cliques in the network, we know that for any $w \in \mathcal{W}^{\Pi}$ the maximum reward rate $R(w)$ is achieved by some trunk reservation strategy. Occasionally, we will therefore refer to the corresponding bounds as 'trunk reservation' bounds.

Note that we cannot determine $\lambda_{\mathcal{W}^{\Pi}}$ by solving either of the above two linear programs directly, since there are an infinite number of inequalities (variables in the dual version) involved. From linear programming theory, however, we know that at most a finite number of these are relevant. We now describe two approaches to obtain $\lambda_{\mathcal{W}^{\Pi}}$ exploiting that fact.

In the first approach, we generate a finite yet exhaustive subset including all the relevant inequalities. For any subset $\mathcal{J} \subseteq \mathcal{I}$, denote $\mathcal{P}^{\mathcal{J}} := \{x \in \mathcal{R}_+^{\mathcal{J}} : \sum_{i \in \mathcal{I}} w_i x_i \leq R(w)$ for all $w \in \mathcal{R}_+^{\mathcal{J}}\}$. By definition, $\lambda_{\mathcal{W}^{\Pi}}$ may be obtained by maximizing $\sum_{i \in \mathcal{I}} x_i$ subject to the constraints $(x_i)_{i \in \mathcal{J}} \in \mathcal{P}^{\mathcal{D}}$ for all $\mathcal{D} \in \Pi$. Also, define $\mathcal{A}^{\mathcal{J}}$ as the convex hull of the carried traffic combinations in the subnetwork of the cells $i \in \mathcal{J}$ achievable by the class of stationary deterministic admission policies. Observe that the convex hull is a polytope, since there are only finitely many stationary deterministic admission policies.

Lemma 3.3: For any subset $\mathcal{J} \subseteq \mathcal{I}$,

$$\mathcal{A}^{\mathcal{J}} = \mathcal{P}^{\mathcal{J}}.$$

Proof: The inclusion to the right is implied by the definition of $R(w)$. The inclusion to the left holds by virtue of the optimality of the class of stationary deterministic admission policies. □

The above lemma implies that $\lambda_{\mathcal{W}^{\Pi}}$ may be obtained by maximizing $\sum_{i \in \mathcal{I}} x_i$ subject to the constraints $(x_i)_{i \in \mathcal{J}} \in \mathcal{A}^{\mathcal{D}}$ for all $\mathcal{D} \in \Pi$. Thus, it suffices to generate the set of facet-defining inequalities of the polytopes $\mathcal{A}^{\mathcal{D}}$ for all $\mathcal{D} \in \Pi$.

In the second approach, we identify the subset of relevant inequalities more indirectly. In the dual formulation, it is quite natural to interchange the roles of the coefficients w and the variables $y(w)$. For example, fixing $y(w) = 1$ for all $w \in \mathcal{W}^*$, we find that $\sum_{w \in \mathcal{W}^*} R(w) \geq \mu_{\mathcal{W}^*} \geq \mu_{\mathcal{W}^{\Pi}}$ for any subset $\mathcal{W}^* \subseteq \mathcal{W}^{\Pi}$ with the property that $\sum_{w \in \mathcal{W}^*} w_i \geq 1$ for all $i \in \mathcal{I}$. The next theorem establishes that this in fact holds with equality for subsets $\mathcal{W}^*$ of remarkably small size.

Theorem 3.4: For any set Π, the optimum value $\lambda_{\mathcal{W}^{\Pi}} = \mu_{\mathcal{W}^{\Pi}}$ equals the optimum value V^{Π} of the following convex programming problem

$$\min \sum_{\mathcal{D} \in \Pi} R(w^{\mathcal{D}}) \tag{12}$$

$$\text{sub} \sum_{\mathcal{D} \in \Pi} w_i^{\mathcal{D}} = 1 \quad \text{for all } i \in \mathcal{I} \tag{13}$$

$$w^{\mathcal{D}} \in \mathcal{R}_+^{\mathcal{D}} \quad \text{for all } \mathcal{D} \in \Pi. \tag{14}$$

Proof: We first prove that $V^{\Pi} \geq \mu_{\mathcal{W}^{\Pi}}$. Let $\{v^{\mathcal{D}}\}_{\mathcal{D} \in \Pi}$ be the optimal solution to the problem (12)–(14), so $V^{\Pi} = \sum_{\mathcal{D} \in \Pi} R(v^{\mathcal{D}})$. The statement preceding the theorem then indicates that $\sum_{\mathcal{D} \in \Pi} R(v^{\mathcal{D}}) \geq \mu_{\mathcal{W}^{\Pi}}$.

We now prove that $\mu_{\mathcal{W}^{\Pi}} \geq V^{\Pi}$. Let $\{z(w)\}_{w \in \mathcal{W}^{\Pi}}$ be the optimal solution to the dual problem (9)–(11), so $\mu_{\mathcal{W}^{\Pi}} = \sum_{w \in \mathcal{W}^{\Pi}} z(w) R(w)$. From optimality, we may conclude that the $z(w)$'s satisfy the constraints (10) with strict equality, since $R(\cdot)$ is an increasing function.

Let $z^{\mathcal{D}}(w) \geq 0$ be variables such that $\sum_{\mathcal{D} \in \Pi} z^{\mathcal{D}}(w) = z(w)$ for all $w \in \mathcal{W}$ and $z^{\mathcal{D}}(w) = 0$ if $w \notin \mathcal{R}_+^{\mathcal{D}}$. Now define $v^{\mathcal{D}} := \sum_{w \in \mathcal{W}^{\Pi}} z^{\mathcal{D}}(w) w$ for all $\mathcal{D} \in \Pi$. It is easily verified that $\{v^{\mathcal{D}}\}_{\mathcal{D} \in \Pi}$ satisfies the constraints (13)–(14). Plugging the $v^{\mathcal{D}}$'s into the objective function (12) then gives $\sum_{\mathcal{D}\in\Pi} R(v^{\mathcal{D}}) \geq V^{\Pi}$.

It remains to be shown that $\sum_{w\in\mathcal{W}^{\Pi}} z(w)R(w) \geq \sum_{\mathcal{D}\in\Pi} R(v^{\mathcal{D}})$. Note that

$$\sum_{w\in\mathcal{W}^{\Pi}} z(w)R(w) = \sum_{w\in\mathcal{W}^{\Pi}} \sum_{\mathcal{D}\in\Pi} z^{\mathcal{D}}(w)R(w) = \sum_{\mathcal{D}\in\Pi} \sum_{w\in\mathcal{W}^{\Pi}} z^{\mathcal{D}}(w)R(w).$$

Now define $\zeta^{\mathcal{D}} := \sum_{w\in\mathcal{W}^{\Pi}} z^{\mathcal{D}}(w)$ for all $\mathcal{D}\in\Pi$. Using the fact that $R(\cdot)$ is a convex function, and that $R(\theta w) = \theta R(w)$ for any scalar $\theta \geq 0$, we obtain

$$\sum_{w\in\mathcal{W}^{\Pi}} z^{\mathcal{D}}(w)R(w) = \zeta^{\mathcal{D}} \sum_{w} \frac{z^{\mathcal{D}}(w)}{\zeta^{\mathcal{D}}} R(w) \geq \zeta^{\mathcal{D}} R\left(\frac{\sum_w z^{\mathcal{D}}(w)w}{\zeta^{\mathcal{D}}}\right) = \zeta^{\mathcal{D}} R\left(\frac{v^{\mathcal{D}}}{\zeta^{\mathcal{D}}}\right) = R(v^{\mathcal{D}}).$$

□

We now revisit the examples studied in Section II to illustrate how the revenue-based bounds may be used to improve upon the Erlang bound.

Example 2.1 (Cont'd): We first return to the three-cell linear network of Example 2.1. From Theorems 3.1 and 3.4, we conclude that the carried traffic is bounded above by $\min_{0\leq y\leq 1} V(y)$, with $V(y) = R(\nu_1,\nu_2,\nu_3;1,y,0) + R(\nu_1,\nu_2,\nu_3;0,1-y,1)$.

Since the function $R(\cdot)$ is convex in the reward vector, the function $V(\cdot)$ is convex as well. Hence, if $\nu_1 = \nu_3$, symmetry arguments imply that $V(y)$ is minimal for $y = 1/2$. For $C = 10$, $\nu_1 = \nu_2 = \nu_3 = 5$, we obtain an upper bound of 12.00 on carried traffic, which corresponds to a lower bound $\bar{B} \approx 0.200$ on blocking, tightening the Erlang bound.

Now suppose the offered traffic is $(\nu_1,\nu_2,\nu_3) = (4,5,6)$. The Erlang bound then yields $\bar{B} \approx 0.192$. Using numerical optimization, we find that $V(y)$ achieves its minimum value 11.77 for $y \approx 0.295$, which produces the bound $\bar{B} \approx 0.215$. The minimum achievable blocking is in fact $\bar{B} \approx 0.225$. □

Example 2.2 (Cont'd): We now return to the four-cell planar network of Example 2.2. Remember that the two center cells may be lumped together so that the network reduces to that of Example 2.1. Hence, the carried traffic is bounded above by $\min_{0\leq y\leq 1} V(y)$, with $V(y) = R(\nu_1,\nu_2+\nu_3,\nu_4;1,y,0) + R(\nu_1,\nu_2+\nu_3,\nu_4;0,1-y,1)$. If $\nu_1 = \nu_4$, then symmetry arguments imply again that $V(y)$ is minimal for $y = 1/2$. For $C = 15, \nu_1 = \nu_2 = \nu_3 = \nu_4 = 5$, we obtain an upper bound of 16.58 on carried traffic, which corresponds to a lower bound $\bar{B} \approx 0.171$ on blocking, tightening the Erlang bound.

Now suppose the offered traffic is $(\nu_1,\nu_2,\nu_3,\nu_4) = (5,3,5,7)$. The Erlang bound then yields $\bar{B} \approx 0.135$. We find that $V(y)$ achieves its minimum value 17.03 for $y \approx 0.320$, which produces the bound $\bar{B} \approx 0.149$. The minimum achievable blocking is in fact $\bar{B} \approx 0.160$. □

IV. Symmetric Networks

We now focus on symmetric, possibly infinite, networks. We may then impose the constraint that the carried traffic be equal in each cell, without affecting the maximum achievable amount of carried traffic. Thus, adding the constraint $x_i = z$ for all $i \in \mathcal{I}$ to the linear program of Theorem 3.1, we find that for any set $\mathcal{D} \subseteq \mathcal{I}$, the maximum average amount of carried traffic per cell is bounded above by

$$z^{\mathcal{D}} = \min_{w\in\mathcal{R}_+^{\mathcal{D}}} \frac{R(w)}{\sum_{i\in\mathcal{D}} w_i}. \tag{15}$$

This may in fact also be concluded from Theorem 3.4, using symmetry arguments.

Since $R(\theta w) = \theta R(w)$ for any scalar $\theta \geq 0$, we may also impose the constraint $\sum_{i\in\mathcal{D}} w_i = 1$ in the minimization in (15). Thus,

$$z^{\mathcal{D}} = \min_{w\in\mathcal{U}_+^{\mathcal{D}}} R(w) \tag{16}$$

with $\mathcal{U}_+^{\mathcal{D}} := \{w \in \mathcal{R}_+^{\mathcal{D}}\colon \sum_{i\in\mathcal{D}} w_i = 1\}$.

Obviously, for any set $\mathcal{D} \subseteq \mathcal{I}$, the maximum average amount of carried traffic per cell is also bounded above by

$$\lambda^{\mathcal{D}} = \max\{\lambda\colon (\lambda,\ldots,\lambda) \in \mathcal{A}^{\mathcal{D}}\} \tag{17}$$

with $\mathcal{A}^{\mathcal{D}}$ the achievable carried traffic region for the subnetwork of cells indexed by $\mathcal{D}$. Thus, determining $\lambda^{\mathcal{D}}$ amounts to maximizing the carried traffic subject to the constraint that it be equal in each cell. This is a Markov decision problem with side-constraints, which may be solved using linear programming techniques.

The question naturally arises how the bounds (15) and (17) are related. Notice that (15) may be rewritten as

$$\lambda^{\mathcal{D}} = \max\{\lambda\colon (\lambda,\ldots,\lambda) \in \mathcal{P}^{\mathcal{D}}\} \tag{18}$$

with $\mathcal{P}^{\mathcal{D}}$ as defined in the previous section. Lemma 3.3 saying that $\mathcal{A}^{\mathcal{D}} = \mathcal{P}^{\mathcal{D}}$ then implies that the bounds are identical. As a side-result, we find that the reward-minimizing vector $w^{\mathcal{D}} = \arg\min_{w\in\mathcal{U}_+^{\mathcal{D}}} R(w)$ may be interpreted as the reward vector for which carrying equal amounts of traffic in each cell maximizes the reward rate.

Denote $\Lambda^{\mathcal{D}} := (\lambda^{\mathcal{D}},\ldots,\lambda^{\mathcal{D}})$. Notice that $\Lambda^{\mathcal{D}}$ is the intersection point of the line $\kappa(1,\ldots,1)$ with a facet of the achievable carried traffic polytope $\mathcal{A}^{\mathcal{D}}$. As a rule, $\Lambda^{\mathcal{D}}$ lies in the *interior* of a facet. In that case, the facet is induced by the inequality $\sum_{i\in\mathcal{D}} w_i^{\mathcal{D}} x_k \leq R(w^{\mathcal{D}})$, so $w^{\mathcal{D}}$ is the unique reward-minimizing vector with $\sum_{i\in\mathcal{D}} w_i^{\mathcal{D}} = 1$, and any policy corresponding to a vertex of the facet achieves the reward $R(w^{\mathcal{D}})$. Occasionally, $\Lambda^{\mathcal{D}}$ may be a vertex of the polytope. In that case, any vector $w^{\mathcal{D}}$ with $\sum_{i\in\mathcal{D}} w_i^{\mathcal{D}} = 1$ for which the inequality $\sum_{i\in\mathcal{D}} w_i^{\mathcal{D}} x_i \leq \lambda^{\mathcal{D}}$ is valid for the polytope $\mathcal{A}^{\mathcal{D}}$ is a reward-minimizing vector, and the policy corresponding to the vertex is the unique optimal one for all these $w^{\mathcal{D}}$'s.

The above results may be generalized to the case where the network is not strictly symmetric, but where the cells may still be partitioned into a number of symmetry classes, say M. In these cases, for any set $\mathcal{D} \subseteq \mathcal{I}$, let $\mathcal{D}_m$ index the cells in

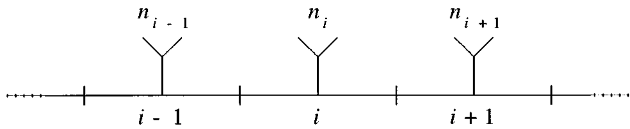

Fig. 4. Linear array of cells.

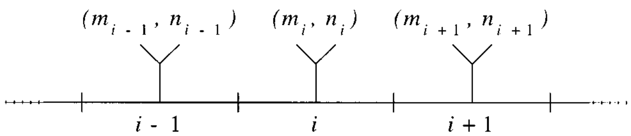

Fig. 5. Linear array of cells with varying reuse.

$\mathcal{D}$ belonging to the m-th symmetry class. The maximum average amount of carried traffic per cell is then bounded above by $\min_{w_m \in \mathcal{U}_+^{\mathcal{D}_m}} R(w_1; \ldots; w_M)$.

We now consider two examples.

Example 4.1: Doubly-Infinite Strip: Consider a similar linear array as in Example 2.1, but now a doubly-infinite strip of cells, instead of just three, as shown in Fig. 4. Each cell is offered traffic at rate ν.

The maximum average carried traffic per cell is bounded above, for any subnetwork of K consecutive cells, by $\min_{w \in \mathcal{U}_+^K} R(w)$, with $\mathcal{U}_+^K := \{w \in \mathcal{R}_+^K : \sum_{k=1}^K w_k = 1\}$. Because of symmetry and convexity, $R(w_1, \ldots, w_K) = (R(w_1, \ldots, w_K) + R(w_K, \ldots, w_1))/2 \geq R((w_1 + w_K)/2, \ldots, (w_1 + w_K)/2)$. Thus, the minimization may be restricted to the set $\{w \in \mathcal{R}_+^K \colon \sum_{k=1}^K w_k = 1, w_1 = w_K, w_2 = w_{K-1}, \ldots\}$. □

Example 4.2: Doubly-Infinite Strip with Varying Reuse: Consider a similar doubly-infinite strip as in Example 4.1, but now a scenario with varying reuse, as illustrated in Fig. 5. Each cell i is partitioned into an inner region $(i, 1)$ and an outer region $(i, 2)$. Each of the inner regions and each of the outer regions is offered traffic at rate $\nu_1 = \alpha\nu$ and $\nu_2 = (1-\alpha)\nu$, respectively. Calls in two different inner regions may always share a channel, while calls in outer regions cannot share a channel with any call in the two adjacent cells.

The maximum average carried traffic per cell is bounded above, for any subnetwork of K_1 inner regions and K_2 outer regions, by $\min_{\substack{w_1 \in \mathcal{U}_+^{K_1} \\ w_2 \in \mathcal{U}_+^{K_2}}} R(w_1; w_2)$. □

V. Numerical Results

A. Doubly-Infinite Strip

We return to the doubly-infinite strip of Example 4.1. Using the two-cell clique constraints, the Erlang bound yields $\bar{B} = \mathrm{Erl}(2\nu; C)$. Adding the single-cell clique constraints does not strengthen the bound.

Let us now turn to the reward bounds. If we consider just a two-cell subnetwork (i.e. a clique), then the reward bound coincides with the Erlang bound. Taking a three-cell subnetwork, we obtain $\lambda_{\max} \leq \min_{y \in [0, 1/2]} R(y, 1 - 2y, y)$. Notice that the calls in the inner cell put higher demands on the network resources. This suggests that we should put higher reward on carrying them if we wish to minimize the maximum achievable reward rate. Indeed, it may be shown that the minimizing reward satisfies $y^* \leq 1/3$. Thus, the minimization may actually be restricted to the interval $y \in [0, 1/3]$.

If we consider a four-cell subnetwork, then $\lambda_{\max} \leq \min_{y \in [0, 1/2]} R(y, 1/2 - y, 1/2 - y, y)$. In this case, the minimization may be confined to the interval $y \in [0, 1/4]$. The convexity properties allow for a simple numerical optimization using Golden-section search. Taking a five-cell or larger subnetwork would generally involve solving a convex programming problem in more than one dimension.

We have performed numerical experiments to compare the bounds with the performance of MP and that of Fixed Channel Assignment (FCA). MP always accepts calls as long as the clique constraints remain satisfied. The blocking for MP is computed using the exact analytical results obtained in [7]. The results for $C = 10$ channels are shown in Fig. 6.

Fig. 6 confirms that MP may substantially reduce blocking over FCA, which may correspond to considerable capacity gains at a given target blocking level. In contrast to the Erlang bound, the reward bounds closely approach the performance of MP. This suggests that the reward bounds are extremely tight. Also, no DCA scheme, however sophisticated, will be able to achieve capacity gains that are significantly larger than those obtained by MP.

It is interesting to investigate how the reward-minimizing y^* varies with the offered traffic ν. Fig. 7 shows the value of y^* as a function of ν for a subnetwork of three cells with $C = 10$ channels. The qualitative behavior may be understood from the interpretation given in the previous section. The vector $(y^*, 1 - 2y^*, y^*)$ is determined by the slope of the facet of the polytope $\mathcal{A}^{\{1,2,3\}}$ that contains the intersection point $(\lambda^*, \lambda^*, \lambda^*)$. As ν varies, the facets of the polytope gradually shift. The smooth segments in the curve reflect the continuous change in the slope of the facet that contains the point $(\lambda^*, \lambda^*, \lambda^*)$. The breaking points in the graph occur when the intersection point occasionally shifts from one facet to another, in which case y^* is not uniquely determined.

In particular, in heavy traffic as $\nu \to \infty$, the achievable traffic polytope approaches the set $\{(\lambda_1, \lambda_2, \lambda_3) \in \mathcal{R}_+^3 : \lambda_1 + 2\lambda_2 + \lambda_3 \leq 2C\}$. Hence, $y^* \to 0.25$ as $\nu \to \infty$. Similarly, it may be verified that $y^* \downarrow 0$ in light traffic as $\nu \downarrow 0$.

B. Interpretation of the Optimal Rewards

It may be helpful to again think of the interpretation of the optimal rewards in the context of the above example. Let x be the traffic carried in each of the two border cells, and let $H(x)$ be the maximum traffic that can be carried in the center cell.

Now consider the optimization problem

$$\begin{array}{ll} \max & x \\ \text{sub} & x \leq H(x) \end{array}$$

i.e., maximize the traffic carried in each of the two border cells subject to the constraint that it not exceed the traffic carried in the center cell. Note that the solution occurs at $x^* = H(x^*)$. Since the function $H(\cdot)$ is concave, the Strong Lagrangean Principle applies, see Whittle [17]. Now form the Lagrangean $L(x, y) = x + (1 - 2y)(H(x) - x)$, associating a multiplier

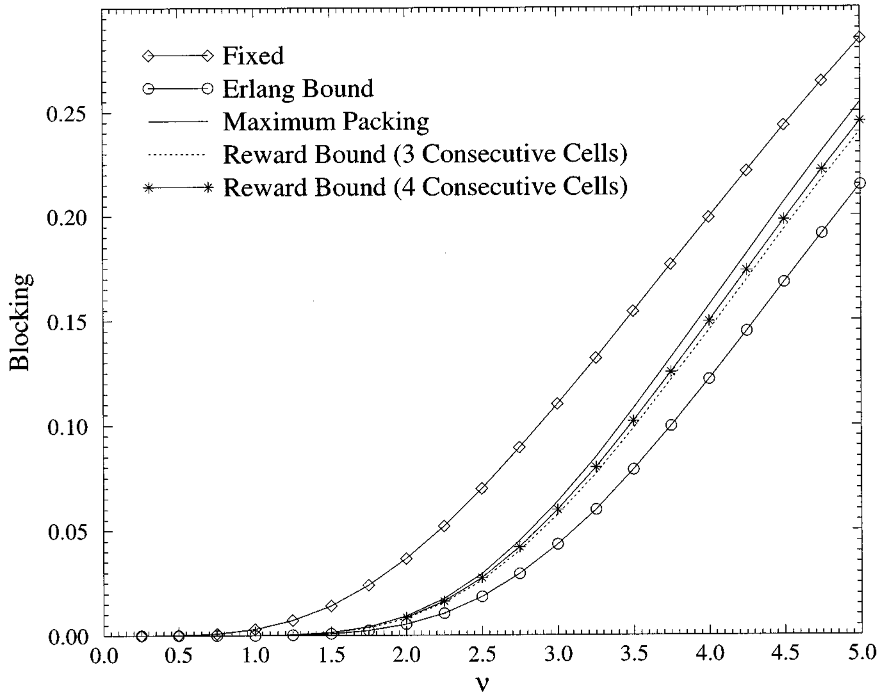

Fig. 6. Erlang bound, reward bounds, and performance of FCA and MP on the doubly-infinite strip as a function of offered traffic for $C = 10$ channels.

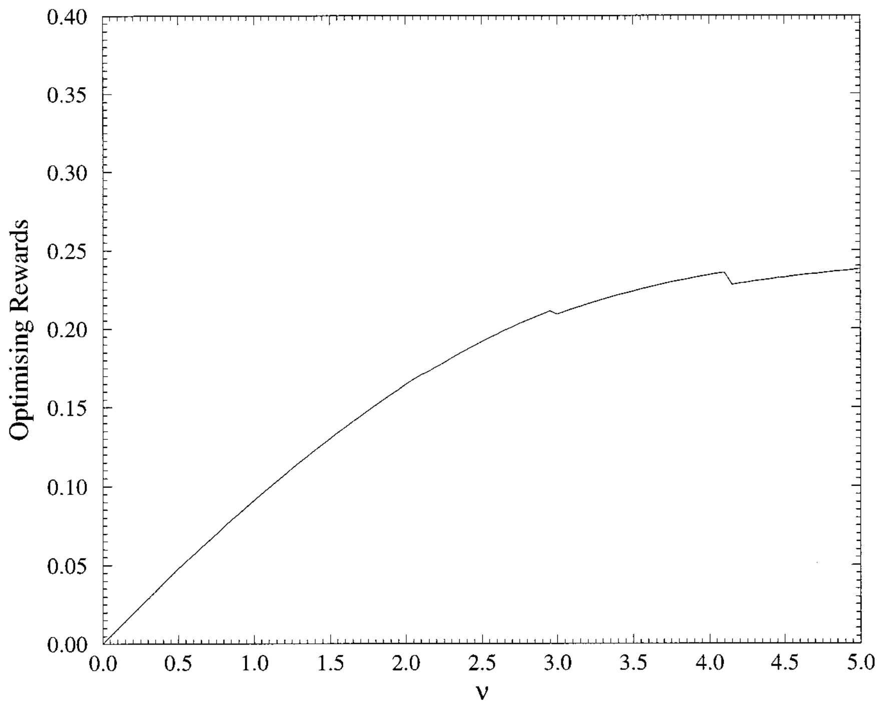

Fig. 7. Value of y^* as a function of offered traffic ν for a subnetwork of 3 consecutive cells with $C = 10$ channels.

$(1-2y)$ with the constraint $x \le H(x)$. The dual problem is then, see theorem 3.11 on page 61 of Whittle [17]

$$\begin{aligned}\min_y L(x,y) &= \min_y \max_x [2xy + (1-2y)H(x)] \\ &= \min_y R(y, 1-2y, y)\end{aligned}$$

and the minimum is achieved at the solution to the primal problem. We thus see that the use of the reward vector in identifying the tightest constraint on the achievable carried traffic region is an application of the duality principle of mathematical programming.

C. *Infinite Hexagonal Grid*

Consider a similar hexagonal network as in Example 2.2, but now an infinite grid, instead of just four cells. Each cell is offered traffic at rate ν.

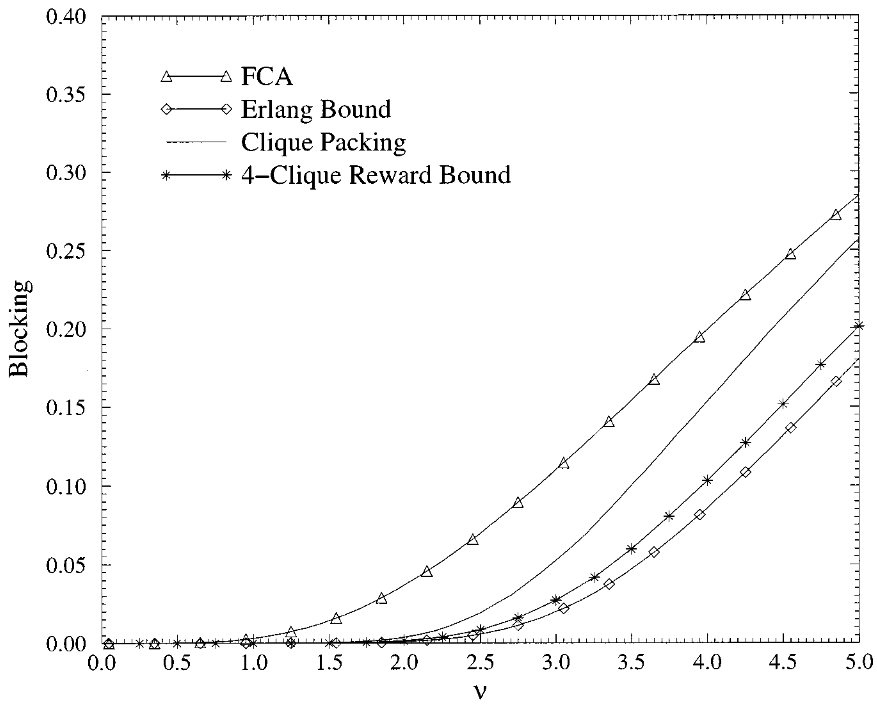

Fig. 8. Erlang bound, four-cell reward bound, and performance of FCA and CMP as a function of offered traffic on an infinite hexagonal grid with $C = 15$ channels.

Using the three-cell clique constraints, the Erlang bound yields $\bar{B} = \mathrm{Erl}(3\nu; C)$. Adding the single-cell clique constraints does not strengthen the bound.

Let us now move to the reward bounds. If we consider just a three-cell subnetwork (i.e. a clique), then the reward bound coincides with the Erlang bound. Taking a four-cell subnetwork as in Fig. 2, we obtain $\lambda_{\max} \leq \min_{y\in[0,1/2]} R(y, 1-2y, y)$, with the center cell offered traffic at double the rate. As before, the minimization may actually be restricted to the interval $y \in [0, 1/3]$. Taking a five-cell or larger subnetwork would generally involve solving a convex programming problem in more than one dimension.

We have conducted numerical experiments to compare the bounds with the performance of Clique Maximum Packing (CMP) and that of FCA. Like in the one-dimensional case, CMP always accepts calls as long as the clique constraints remain satisfied. Other than in the linear case, this may not be sufficient for a feasible assignment of channels to users to exist. Our primary purpose is however to evaluate the reward bounds, which are still valid for CMP. Because there are no exact analytical results available in the planar case, the blocking for CMP is obtained using simulation for a 6×6 wrap-around grid. The results for $C = 15$ channels are shown in Fig. 8.

Fig. 8 demonstrates that also in the planar case, CMP may substantially reduce blocking over FCA. The reward bound still sharpens the Erlang bound, but does not approach the performance of CMP as closely as in the linear case. This discrepancy could in principle be caused by two factors: 1) the reward bound may fail to be tight in the planar case; and 2) CMP may fail to be nearly optimal in the planar case. To resolve this issue, we considered a seven-cell subnetwork which gives $\lambda_{\max} \leq \min_{y\in[0,1/7]} R(y, y, y, y, y, y, 1-6y)$ as an upper bound on carried traffic. To limit the state space of the Markov decision problem, we reduced the number of channels to $C = 6$. The results are displayed in Fig. 9.

Fig. 9 shows that the seven-cell reward bound *does* closely approach the performance of CMP. Thus, CMP in fact continues to be nearly optimal in the planar case, and the discrepancy with the four-cell reward bound mentioned above may be largely attributed to the size of the subnetwork being insufficient.

In the numerical experiments, we have focused on scenarios with relatively small reuse groups and a limited number of channels. In principle, the bounds may also be computed for larger reuse groups or a larger number of channels. However, the calculations may be significantly hampered by the curse of dimensionality in dynamic programming.

VI. Scenarios With Varying Re-Use

A. Doubly-Infinite Strip With Varying Reuse

We return to the doubly-infinite strip with varying reuse of Example 4.2. The Erlang bound no longer applies at the level of cells now, but does still apply at the level of the regions. Considering cliques consisting of two outer regions and one inner region yields the bound $\bar{B} = ((2-\alpha)/2)\,\mathrm{Erl}((2-\alpha)\nu; C)$. Not surprisingly, the bound is decreasing in α, the fraction of traffic offered to the inner regions. Adding the constraints $B_{i,1} \geq \mathrm{Erl}(\alpha\nu; C)$, the bound may be tightened to $\bar{B} = (1/2)[(2-\alpha)\,\mathrm{Erl}((2-\alpha)\nu; C) + \alpha\,\mathrm{Erl}(\alpha\nu; C)]$.

We now turn to the reward bounds. Taking a clique consisting of one inner region and two outer regions, we obtain $\lambda_{\max} \leq R(1; 1/2, 1/2)$ as an upper bound on carried traffic. If we consider a subnetwork consisting of two cliques with a

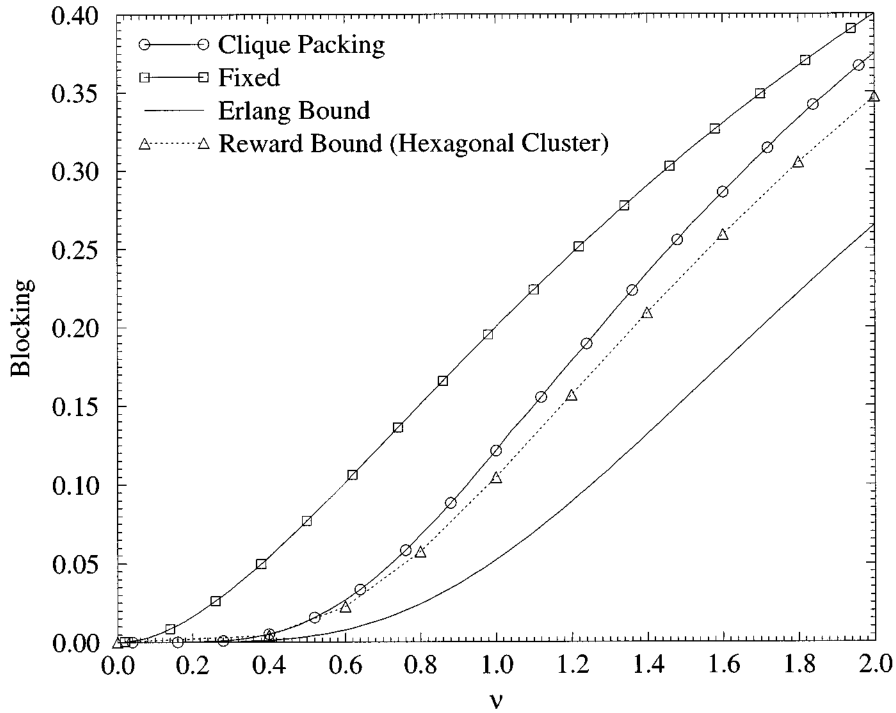

Fig. 9. Erlang bound, seven-cell reward bound, and performance of FCA and CMP as a function of offered traffic on an infinite hexagonal grid with $C = 6$ channels.

common inner region, then $\lambda_{\max} \leq \min_{y \in [0,1/2]} R(1; y, 1-2y, y)$. As before, the convexity properties allow for a simple numerical optimization using Golden-section search. The calculation of $R(\cdot)$ in each iteration, however, is of formidable complexity for all but the smallest number of channels, and is the main obstacle in considering larger subnetworks.

We have performed numerical experiments to compare the bounds with the performance of MP and FCA, both adapted to the varying reuse constraints. In FCA, we statically assign C_1 channels to each of the inner regions, and C_2 channels to each of the outer regions, with $C_1 + 2C_2 = C$. MP always accepts calls as long as the clique constraints remain satisfied (cliques now existing of one inner cell and two outer cells). The blocking for MP is calculated using the exact analytical results obtained in the Appendix.

The results for $C = 10$ channels and a fraction $\alpha = 0.3$ of traffic offered to the inner regions are shown in Fig. 10. For FCA, we plot the minimum blocking over all feasible combinations of (C_1, C_2). (Observe that the optimal combination depends on the offered traffic.)

Fig. 10 indicates that also in the case of varying reuse MP may substantially reduce blocking over FCA. Comparing with Fig. 6, we see that the reduction in blocking is larger than in the case of uniform reuse. In contrast to MP, tighter reuse does not significantly help reduce blocking in FCA. Presumably, the benefits from tighter reuse do not offset the loss in trunking efficiencies from splitting the cells into smaller regions. This suggests that DCA is crucial in fully extracting the potential capacity gains from tighter reuse. Although the reward bounds still improve upon the Erlang bound, they slightly diverge from the performance of MP now. As it turns out, considering larger subnetworks does not significantly help close the gap. To explain these observations, it is helpful to consider the blocking of inner and outer calls separately as displayed in Fig. 11.

Fig. 11 reveals that the blocking of outer calls in MP is about twice that of inner calls for moderate values of blocking. Drawing upon the theory of loss networks, see Kelly [8], the blocking ratio may be understood from the fact that outer calls require a channel in four cliques, whereas inner calls in only two. To maximize carried traffic, however, blocking should be primarily inflicted on the outer calls, since these put higher demands on the network resources. Indeed, to minimize blocking in FCA, more and more channels are shifted from the outer regions to the inner regions as the offered traffic increases, and thus the blocking ratio gets larger and larger, up to the point that all the channels are allocated to the inner regions.

This reflects the inherent tradeoff between efficiency and fairness that arises in the case of varying reuse, see also Shimada *et al.* [14] and Valenzuela [15]. Schemes which minimize blocking intrinsically favor inner calls over outer calls, whereas schemes which do not discriminate among calls inevitably produce higher network-average blocking.

The versions of FCA and MP described above may be viewed as two extreme ways of operating a network with varying reuse. For conciseness, let us refer to the set of all inner regions as the inner layer, and to the outer regions as the outer layer. Two possible intermediate approaches are as follows.

i) Borrowing channels within each cell, but not among cells. We still statically assign C_1 channels to each of the inner regions and C_2 channels to each of the outer regions, with $C_1 + 2C_2 = C$, but allow outer-region channels to be borrowed by inner-region calls (not vice versa). The joint

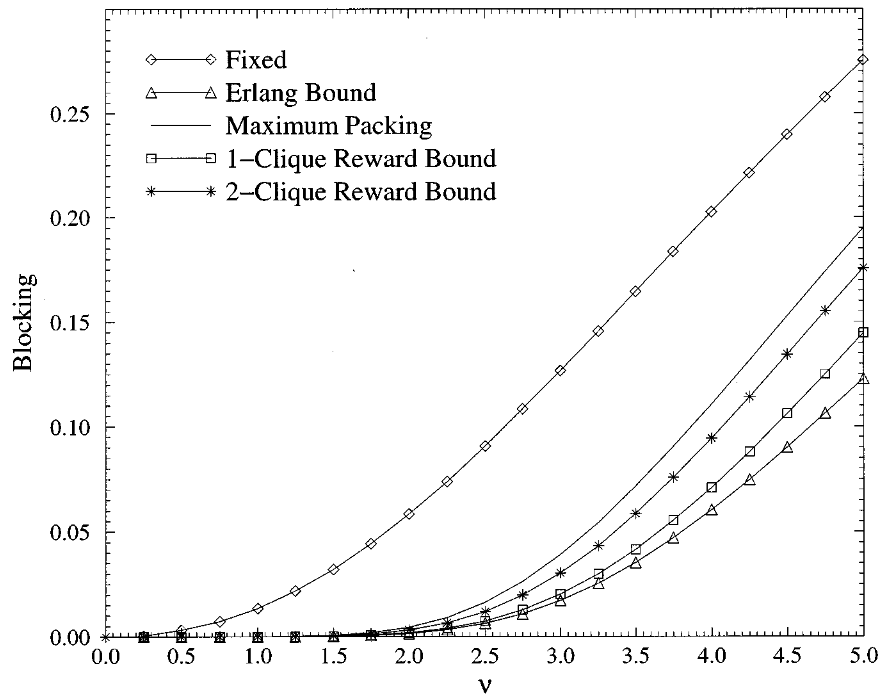

Fig. 10. Erlang bound, reward bounds, and performance of FCA and MP as a function of offered traffic on the doubly-infinite strip with varying reuse and $C = 10$ channels.

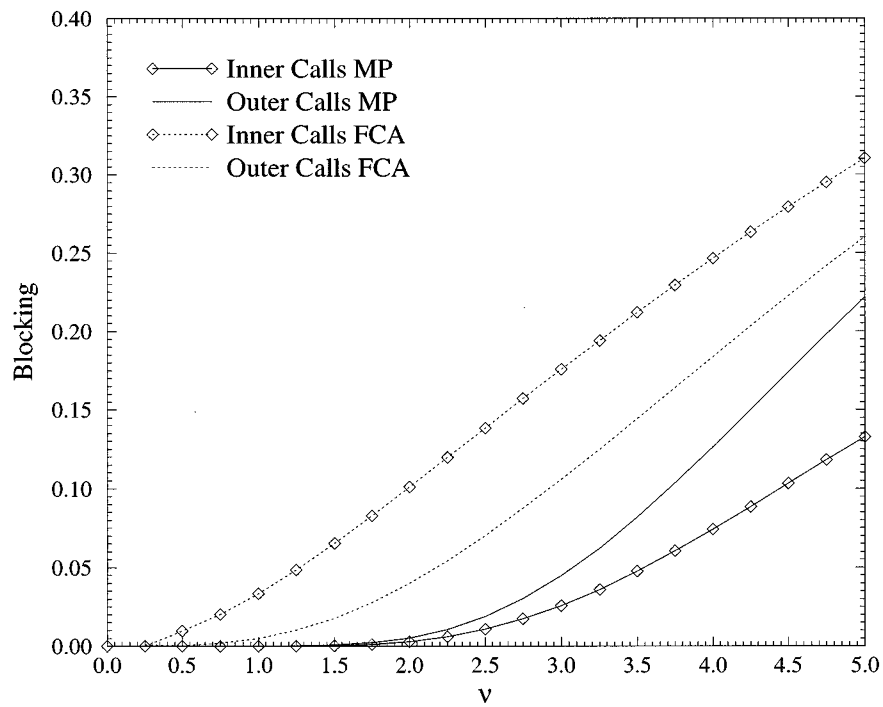

Fig. 11. Blocking of inner and outer calls for FCA and MP as a function of offered traffic on the doubly-infinite strip with varying reuse and $C = 10$ channels.

distribution of the number of inner and outer calls in a particular cell then has the product form

$$\pi(n_1, n_2) = G^{-1}\alpha^{n_1}(1-\alpha)^{n_2}\nu^{n_1+n_2}$$

for all (n_1, n_2) with $n_2 \leq C_2$, $n_1 + n_2 \leq C_1 + C_2$, with G representing the normalization constant. The blocking for inner calls is $B_1 = \sum_{k=C_1}^{C_1+C_2} \pi(k, C_1 + C_2 - k)$. The blocking for outer calls is $B_1 = B_1 + \sum_{k=0}^{C_1-1} \pi(k, C_2)$.

ii) Sharing channels within both the inner and outer layer, but not between these two layers. We now allocate C' channels to the inner layer and C'' channels to the outer layer, with $C' + C'' = C$. The blocking for inner calls is then simply given by $B_1 = \mathrm{Erl}(\alpha\nu; C')$. The blocking B_2 for outer calls satisfies the bounds derived in Example 4.1 for the standard doubly-infinite strip (but now with C'' channels and offered traffic $(1-\alpha)\nu$). (Generally,

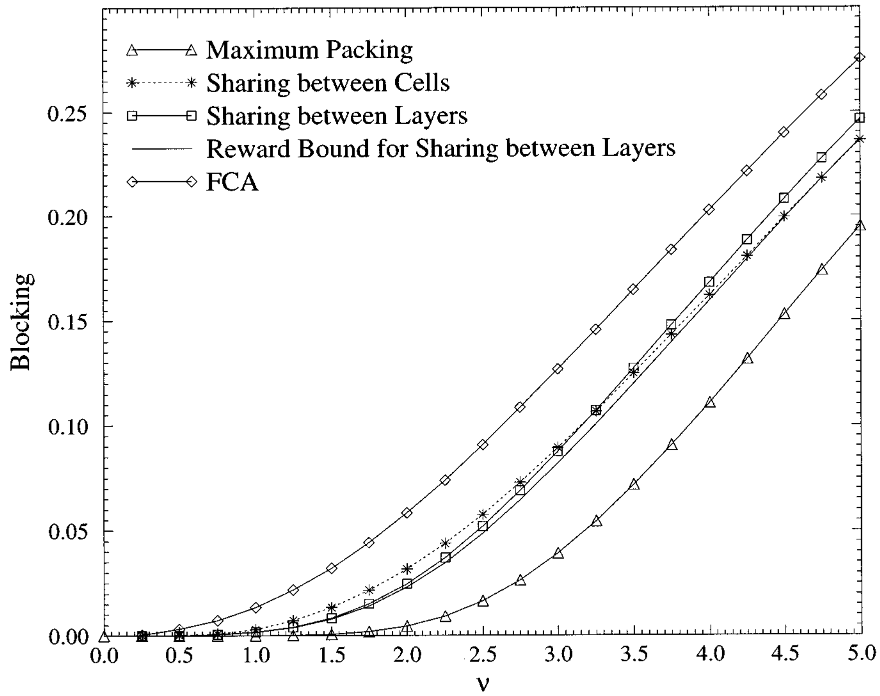

Fig. 12. Performance of FCA, MP, approach A, and approach B as a function of offered traffic on the doubly-infinite strip with varying reuse and $C = 10$ channels.

imposing a hard boundary between the inner and outer layer results in two independent networks with different but fixed reuse factors.)

We have investigated how the performance of these two intermediate approaches compares to that of the two extreme strategies examined before. In case B, we also consider a scenario in which the outer layer is operated using MP. The results for $C = 10$ channels and a fraction $\alpha = 0.3$ of traffic offered to the inner regions are plotted in Fig. 12. We show the minimum blocking over all feasible combinations of (C_1, C_2) and (C', C''), respectively. The figure indicates that the two intermediate approaches for sharing the channels actually perform quite similarly, but that both fall short of MP. This reinforces the earlier statement that unrestricted sharing is crucial in fully exploiting the potential capacity gains from tighter reuse.

B. Infinite Hexagonal Grid With Varying Reuse

Consider a similar infinite hexagonal grid as in the previous section, but now a scenario with varying reuse as described in Example 4.2. Each of the inner regions and each of the outer regions is offered traffic at rate $\nu_1 = \alpha\nu$ and $\nu_2 = (1-\alpha)\nu$, respectively.

Considering cliques consisting of three outer regions and one inner region yields the Erlang bound $\bar{B} = ((3-2\alpha)/3)\,\mathrm{Erl}((3-2\alpha)\nu; C)$. Adding the constraints $B_{i,1} \geq \mathrm{Erl}(\alpha\nu; C)$, the bound may be tightened to $\bar{B} = (1/3)[(3-2\alpha)\,\mathrm{Erl}((3-\alpha)\nu; C) + 2\alpha\,\mathrm{Erl}(\alpha\nu; C)]$.

We now move to the reward bounds. Taking a clique consisting of one inner region and three outer regions, we obtain $\lambda_{\max} \leq R(1; 1/3, 1/3, 1/3)$ as an upper bound on carried traffic. If we consider a subnetwork consisting of two cliques with a common inner region, then $\lambda_{\max} \leq \min_{y \in [0,1/2]} R(1; y, 1-2y, y)$, with traffic offered to the center cell at double the rate.

We have conducted numerical experiments to compare the bounds with the performance of CMP and FCA, both adapted to the varying reuse constraints. In FCA, we statically assign C_1 channels to each of the inner regions, and C_2 channels to each of the outer regions, with $C_1 + 3C_2 = C$. CMP always accepts calls as long as the clique constraints remain satisfied (cliques now existing of one inner region and three outer regions). (As mentioned earlier, this may not be sufficient for a feasible assignment of channels to users to exist, but the reward bounds are still valid for CMP.) In the absence of exact results, the blocking for CMP is obtained using simulation for a 6×6 wrap-around grid.

The results for $C = 15$ channels and a fraction $\alpha = 0.3$ of traffic offered to the inner regions are shown in Fig. 13. For FCA, we plot the minimum blocking over all feasible combinations of (C_1, C_2).

Fig. 13 shows that also in the planar case with varying reuse CMP may substantially reduce blocking over FCA. Although the reward bounds improve upon the Erlang bound, they considerably deviate from the performance of CMP. The discrepancy may be attributed to two sources: i. Like in the planar case with uniform reuse, the reward bounds for four-cell subnetworks fail to be tight. Indeed, the gap may be somewhat reduced by considering larger subnetworks, which may however prove extremely demanding; ii. Like in the linear case with varying reuse, CMP fails to be nearly optimal. As observed before, CMP favors inner calls over outer calls, but not to the extent required to maximize carried traffic. This is illustrated in Fig. 14.

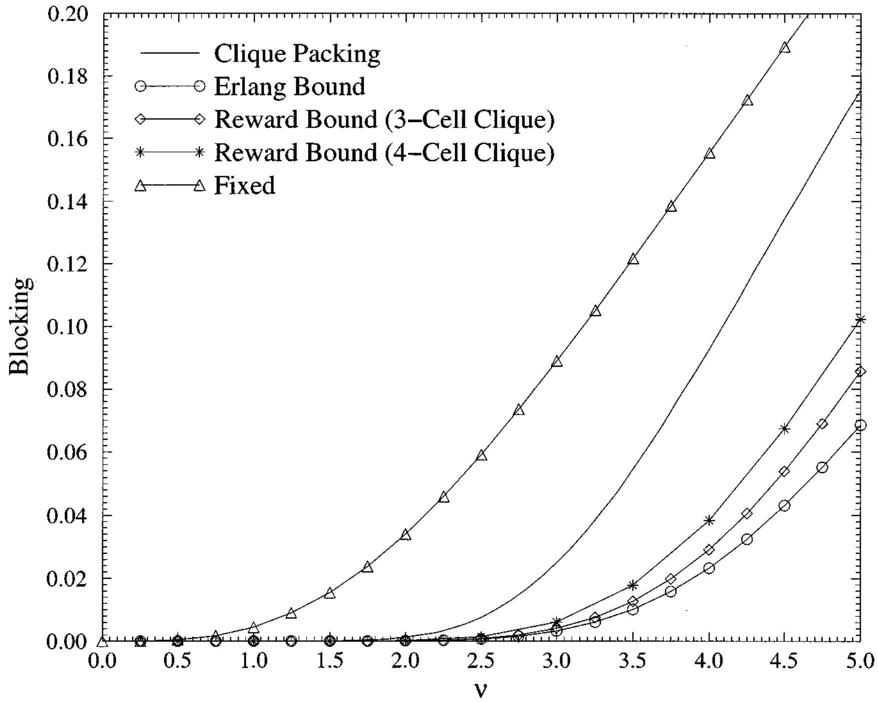

Fig. 13. Erlang bound, reward bounds, and performance of FCA and MP as a function of offered traffic on an infinite hexagonal network with varying reuse and $C = 15$ channels.

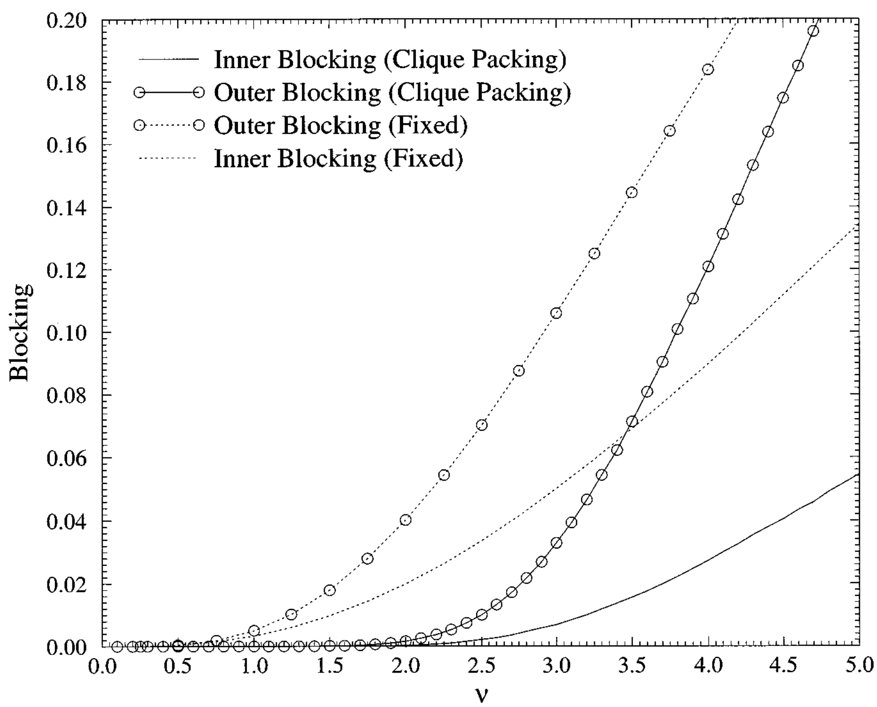

Fig. 14. Blocking of inner and outer calls for FCA and CMP as a function of offered traffic on an infinite hexagonal network with varying reuse and $C = 15$ channels.

VII. Asymptotic Analysis

We now further investigate the tradeoff between efficiency and fairness that arises in the case of varying reuse. We focus on the doubly-infinite strip with uniform offered traffic of Example 4.2. We consider a scenario in which the number of channels and the offered traffic grow large in proportion to one another, i.e., $C \to \infty$, $\nu \to \infty$, and $\nu/C = \rho$. Note that $\mathrm{Erl}(\rho C; C) \to \max\{1 - 1/\rho, 0\}$ as $C \to \infty$ for any $\rho \geq 0$.

Denote by B_1 and B_2 the blocking of inner and outer calls, respectively. Denote by λ_1 and λ_2 the carried traffic in each of the inner regions and outer regions, respectively. By definition, $\lambda_1 = (1 - B_1)\nu_1$, $\lambda_2 = (1 - B_2)\nu_2$. Note that $\bar{B} = \alpha B_1 + (1 - \alpha)B_2 = (\nu_1 B_1 + \nu_2 B_2)/\nu = 1 - (\lambda_1 + \lambda_2)/\nu$.

Considering a clique of one inner cell and two outer cells, we have

$$\lambda_1 + 2\lambda_2 \leq R(1;1,1) \tag{19}$$

$$\lambda_1 \leq \alpha\nu \tag{20}$$

$$\lambda_2 \leq (1-\alpha)\nu. \tag{21}$$

Observe that $R(1;1,1) \leq \min\{(2-\alpha)\nu, C\}$. Maximizing $\lambda_1 + \lambda_2$ subject to the constraints (19)–(21), we find that $\bar{B} \geq \beta^*$, with

$$\beta^* = \begin{cases} 0 & \rho \leq \dfrac{1}{2-\alpha} \\ 1 - \dfrac{1+\alpha\rho}{2\rho} & \dfrac{1}{2-\alpha} \leq \rho \leq \dfrac{1}{\alpha} \\ 1 - \dfrac{1}{\rho} & \rho \geq \dfrac{1}{\alpha} \end{cases}. \tag{22}$$

Define $\gamma := \min\{\alpha\rho, 1\}$. Now suppose we reserve a fraction γ of the channels for the inner calls, and leave the remaining fraction $1-\gamma$ of the channels for the outer calls. Then $B_1 = \text{Erl}(\alpha\rho C; \gamma C)$, $B_2 = \text{Erl}((1-\alpha)\rho C; (1-\gamma)C)$. It is easily verified that $\bar{B}$ approaches β^* as $C \to \infty$, i.e., the bound β^* is asymptotically achievable and hence tight. Observe that this strategy only grants capacity to the outer calls that is essentially not needed by the inner calls. This confirms that schemes which minimize network-average blocking will intrinsically favor inner calls over outer calls.

We now examine what the increase in blocking is if we require the blocking of inner calls and outer calls to be equal. Adding the condition $\lambda_1/\nu_1 = \lambda_2/\nu_2$ to the constraints (19)–(21), before maximizing $\lambda_1 + \lambda_2$, we find that $B_1 = B_2 = \bar{B} \geq \beta^{\#}$, with

$$\beta^{\#} = \begin{cases} 0 & \rho \leq \dfrac{1}{2-\alpha} \\ 1 - \dfrac{1}{(2-\alpha)\rho} & \rho \geq \dfrac{1}{2-\alpha}. \end{cases} \tag{23}$$

Now suppose we allocate a fraction $\alpha/(2-\alpha)$ of the channels to the inner calls, and assign the remaining fraction $(1-\alpha)/(2-\alpha)$ of the channels to the outer calls in each cell. Then $B_1 = \text{Erl}(\alpha\rho C; \alpha C/(2-\alpha))$, $B_2 = \text{Erl}((1-\alpha)\rho C; (1-\alpha)C/(2-\alpha))$. It is easily verified that B_1, B_2, and $\bar{B}$ approach $\beta^{\#}$ as $C \to \infty$, i.e., the bound $\beta^{\#}$ is asymptotically achievable and hence tight.

Define $\delta := \beta^{\#} - \beta^*$. From (22) and (23)

$$\delta = \begin{cases} 0 & \rho \leq \dfrac{1}{2-\alpha} \\ \dfrac{\alpha}{2} + \dfrac{1}{\rho}\left(\dfrac{1}{2} - \dfrac{1}{2-\alpha}\right) & \dfrac{1}{2-\alpha} \leq \rho \leq \dfrac{1}{\alpha} \\ \dfrac{1}{\rho}\left(1 - \dfrac{1}{2-\alpha}\right) & \rho \geq \dfrac{1}{\alpha} \end{cases}.$$

This demonstrates that schemes which do not discriminate among calls inevitably produce higher network-average blocking.

We now analyze the asymptotic performance of MP. The Erlang fixed-point approximation for MP may be constructed as follows:

$$1 - A = \text{Erl}(\alpha\rho CA + 2(1-\alpha)\rho CA^3; C)$$
$$1 - B_1 \approx A^2$$
$$1 - B_2 \approx A^4.$$

This approximation is consistent with the earlier observation that for moderate values of blocking, i.e., $A \approx 1$, the blocking of outer calls is about twice that of inner calls.

Asymptotically,

$$A \to \min\left\{\frac{1}{\alpha\rho A + 2(1-\alpha)\rho A^3}, 1\right\}.$$

Since the Erlang fixed-point approximation is asymptotically exact, see Kelly [8], $\bar{B}^{\text{MP}} \to \alpha G + (1-\alpha)G^2$, with

$$G = \min\left\{\frac{-\alpha\rho + \sqrt{\alpha^2\rho^2 + 8(1-\alpha)\rho}}{4(1-\alpha)\rho}, 1\right\}.$$

It may be verified algebraically that $\beta^* \leq \bar{B}^{\text{MP}} \leq \beta^{\#}$ for all values of α and ρ. Fig. 15 plots the values of β^*, $\beta^{\#}$, and $\bar{B}^{\text{MP}}$ as a function of ρ for $\alpha = 0.3$.

VIII. CONCLUSION

The Erlang bound may not always be tight because it fails to exclude carried traffic combinations which are only feasible if call dropping were permitted. The "trunk reservation" bounds which we introduced are also obtained by considering cliques of cells in the network. The construction of these bounds is based on a reward paradigm as an intuitively appealing way of characterizing the *true* achievable carried traffic region, thus exposing any infeasible combinations that may weaken the Erlang bound. The computational complexity increases somewhat, but the bounds may be readily obtained in planar networks.

Even tighter bounds may be obtained by not considering cliques, but subnetworks of cells in which a channel may be used more than just once. In the case of uniform reuse, the revenue-based bounds then closely approach the performance of MP. This suggests not only that the bounds are extremely tight, but also that no DCA scheme, however sophisticated, will be able to achieve significant capacity gains beyond those obtained from MP. The fact that such tight bounds can be obtained by considering just three or four neighboring cells in the network is striking. For a given subnetwork, no tighter bound can be obtained, since the reward paradigm completely demarcates the achievable carried traffic region.

Subsequently, we considered scenarios with varying reuse which may arise in the case of reuse partitioning techniques, measurement-based DCA schemes, or micro-cellular environments. We showed how the analysis presented in Kelly [7] for MP on a doubly-infinite strip may be generalized. The revenue-based bounds extend to these scenarios with varying reuse, but the computational complexity increases further, which means that only relatively small subnetworks can be considered. In these circumstances, however, the bounds slightly diverge from the performance of MP, which inflicts higher blocking on outer calls than inner calls, but not to the extent required to maximize

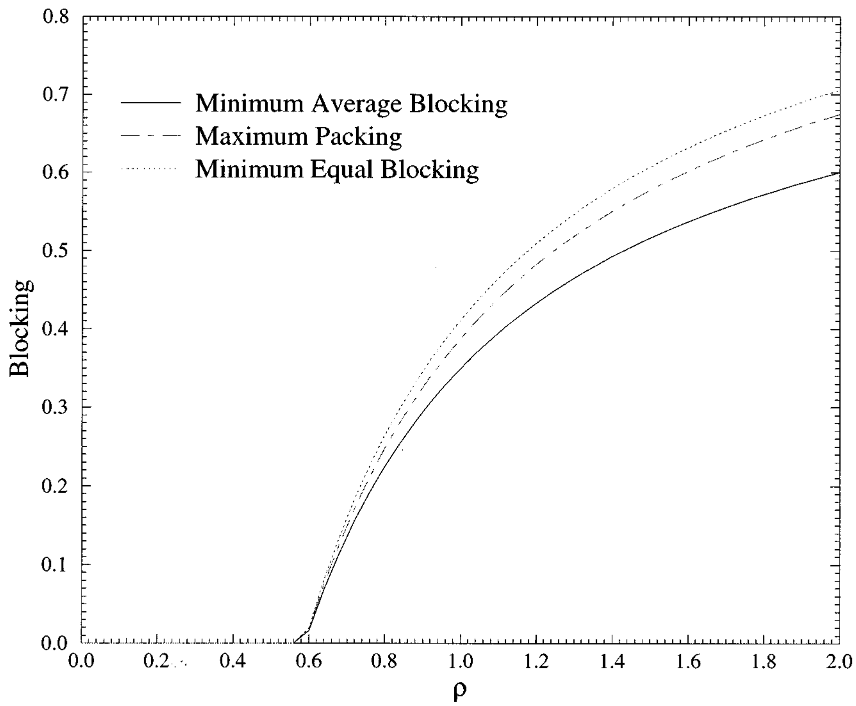

Fig. 15. Minimum average-blocking β^*, minimum equal-blocking $\beta^\#$, and asymptotic performance of MP for varying reuse with a fraction $\alpha = 0.3$ of traffic offered to the inner regions.

carried traffic. This reflects the inherent tradeoff that arises in the case of varying reuse between efficiency and fairness. This observation is consistent with the empirical finding of spatially inhomogeneous blocking in Shimada *et al.* [14] and Valenzuela [15]. Asymptotic analysis confirms that schemes which minimize blocking intrinsically favor inner calls over outer calls, whereas schemes which do not discriminate among calls inevitably produce higher network-average blocking.

In the present paper we have not considered any user mobility. In reality, however, users move around so that calls in progress may occasionally have to be handed off from one cell to another. In the case of varying reuse, hand-off calls may be expected to experience similar high blocking as peripheral calls at set-up. Since calls in progress should in fact receive a preferential treatment, this suggests an even greater need for channel reservation mechanisms in the case of varying reuse.

In the presence of mobility, a more reasonable goal is probably to minimize blocking subject to a dropping constraint, or to minimize a weighted combination of blocking and dropping. The reward paradigm may be generalized to obtain bounds in these cases. The bounds may still be obtained by solving a linear program. However, the complexity of computing the reward coefficients will increase considerably, because the hand-off process is quite complicated. In contrast to fresh calls, hand-off calls do not arrive according to independent Poisson processes. The closer interaction between cells will also dilute the capacity limits that can be lifted from a subnetwork in isolation. Since the bounds may not be as tight, while MP may be far from optimal in view of the need for channel reservation mentioned above, the gap between the two should be expected to increase dramatically.

APPENDIX
BLOCKING FOR MAXIMUM PACKING UNDER VARYING RE-USE

In this Appendix, we extend the analysis presented in Kelly [7] for MP on the doubly-infinite strip to the case of varying reuse described in Example 4.2. We first consider a finite array of $2I + 1$ cells indexed by the set $\mathcal{I} = \{-I, -I+1, \ldots, 0, \ldots, I-1, I\}$. Denote by ν_{i1} and ν_{i2} the offered traffic to the inner and outer region of cell i, respectively. The state of the network may be described by the vector $\mathbf{n} = (\mathbf{n}_i)_{i\in\mathcal{I}} = (m_i, n_i)_{i\in\mathcal{I}}$, with m_i and n_i representing the number of calls in the inner and outer region of cell i, respectively. The set of admissible states of the network is defined as

$$\mathcal{S} = \{\mathbf{n} : n_i + n_{i+1} + \max\{m_i, m_{i+1}\} \le C \text{ for all } \mathcal{C} \in \mathcal{I}\}.$$

Observe that the equilibrium distribution $\pi(\mathbf{n})$ satisfies the detailed balance conditions

$$m_i \pi(\mathbf{n}) = \nu_{i1}\pi(\mathbf{n} - d_i)$$
$$n_i \pi(\mathbf{n}) = \nu_{i2}\pi(\mathbf{n} - e_i)$$

where d_i denotes a vector consisting of all 0's but for a 1 in the position corresponding to the inner region of component vector i. The vector e_i is defined similarly for the outer region of cell i. Thus, the equilibrium distribution is

$$\pi(\mathbf{n}) = G \prod_{i\in\mathcal{I}} \frac{\nu_{i1}^{m_i}}{m_i!}\frac{\nu_{i2}^{n_i}}{n_i!}, \quad \mathbf{n} \in \mathcal{S} \tag{24}$$

with G representing a normalization constant.

We now consider the special case of uniform offered traffic, i.e., $\nu_{i1} = \nu_1, \nu_{i2} = \nu_2$ for all $i \in \mathcal{I}$. Define the *square* matrix $Q(\mathbf{n}, \mathbf{n}')$ by

$$Q((m,n),(m',n')) := \begin{cases} \frac{\nu_1^{m'}}{m'!}\frac{\nu_2^{n'}}{n'!} & n + n' + \max\{m, m'\} \leq C \\ 0 & \text{otherwise.} \end{cases}$$

From (24), we see that the sequence of vectors $\mathbf{n}_i, i \in \mathcal{I}$, determine an inhomogeneous Markov chain, with transition matrix proportional to Q. Indeed, the equilibrium distribution may be written

$$\pi(\mathbf{n}) = \pi_0(\mathbf{n}_0)\Phi(\mathbf{n})$$

with

$$R_I^2\Phi = Q(\mathbf{n}_0, \mathbf{n}_1)\ldots Q(\mathbf{n}_{I-1}, \mathbf{n}_I)Q(\mathbf{n}_0, \mathbf{n}_{-1})\ldots Q(\mathbf{n}_{-I+1}, \mathbf{n}_{-I})$$

$$R_I(\mathbf{n}_0) = \sum_{\mathbf{n}_1', \ldots \mathbf{n}_I'} Q(\mathbf{n}_0, \mathbf{n}_1')\ldots Q(\mathbf{n}_{I-1}', \mathbf{n}_I')$$

and π_0 representing the marginal equilibrium distribution for cell 0. Observe that R_I is simply the sum, over columns, of row $\mathbf{n}$ of Q^I.

Applying the detailed balance conditions to the component vector $\mathbf{n}_0$ for both the inner and outer region, we see that π_0 must satisfy

$$\pi_0(\mathbf{n}_0 + d_0) = \frac{\nu_1}{m_0 + 1}\frac{[R_I(\mathbf{n}_0 + d_0)]^2}{[R_I(\mathbf{n}_0)]^2}\pi_0(\mathbf{n}_0)$$

$$\pi_0(\mathbf{n}_0 + e_0) = \frac{\nu_2}{n_0 + 1}\frac{[R_I(\mathbf{n}_0 + e_0)]^2}{[R_I(\mathbf{n}_0)]^2}\pi_0(\mathbf{n}_0).$$

That is, the marginal equilibrium distribution is similar to what it would be for cell 0 in isolation, but with the birth rates modified by R_I.

Using standard arguments from the Perron-Frobenius theory of nonnegative matrices, we find that as $I \rightarrow \infty$

$$\frac{[R_I(\mathbf{n}_0 + d_0)]^2}{[R_I(\mathbf{n}_0)]^2} \rightarrow \frac{r^2(\mathbf{n}_0 + d_0)}{r^2(\mathbf{n}_0)},$$

$$\frac{[R_I(\mathbf{n}_0 + e_0)]^2}{[R_I(\mathbf{n}_0)]^2} \rightarrow \frac{r^2(\mathbf{n}_0 + e_0)}{r^2(\mathbf{n}_0)}$$

where r denotes the right eigenvector of the matrix Q. Thus, we deduce that the marginal form of the equilibrium distribution for cell 0 is

$$\pi(\mathbf{n}_0) \propto \frac{\nu_1^{m_0}}{m_0!}\frac{\nu_2^{n_0}}{n_0!}r^2(\mathbf{n}_0). \tag{25}$$

This result generalizes to similar linear networks with a larger number of regions within cells. Numerical computation of the marginal distribution for larger problems is impeded by the need to find the Perron-Frobenius eigenvector of increasingly larger matrices.

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