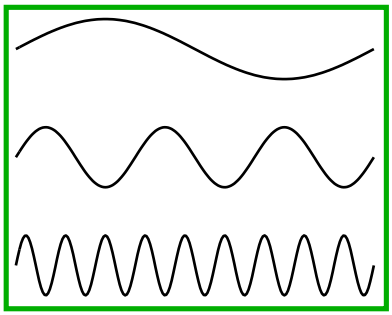
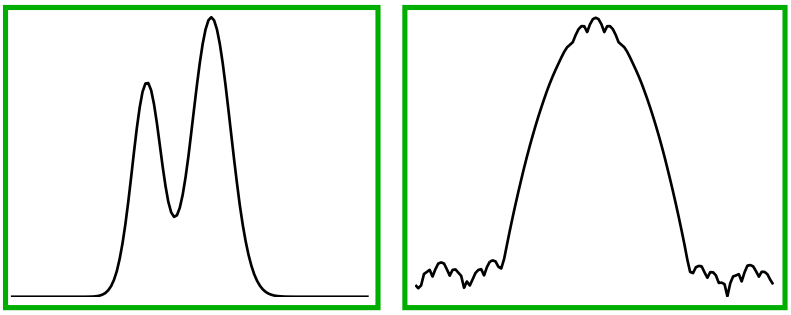
Analyzing Signals

Fourier transform

- frequency content
- linear combination of sin(ωt) and cos(ωt)

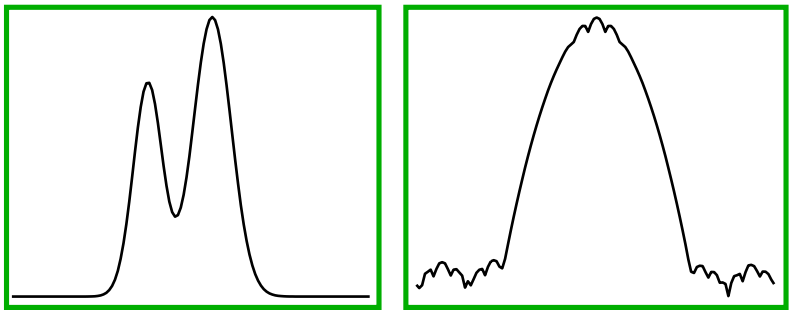


Spectrum



spatial domain frequency domain

Spectrum



spatial domain frequency domain

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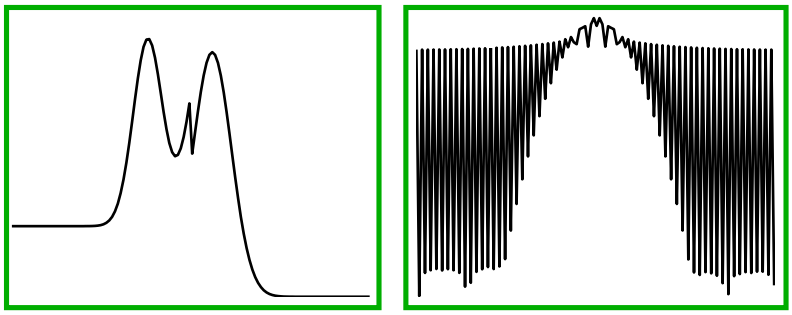
Spectrum



spatial domain frequency domain

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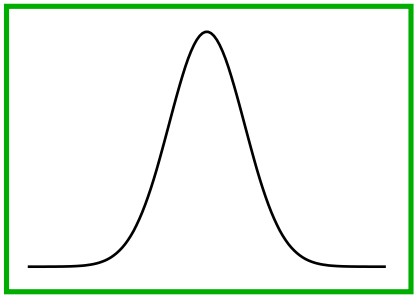
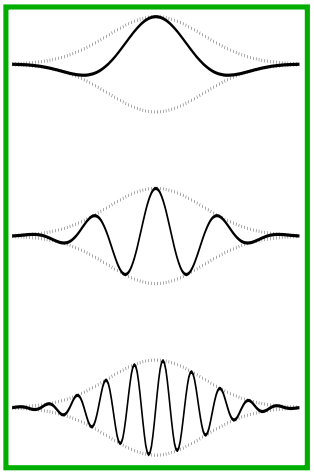
Localized Analysis

Gabor (1940)

- time frequency analysis
- windowed Fourier transform





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Gabor Transform

Find

- frequency ω in the vicinity of **b**

function to analyze

$$F(b,\omega) = \int f(x)g(x-b)\sin(\omega x)dx$$

window function at **b**

at frequency ω

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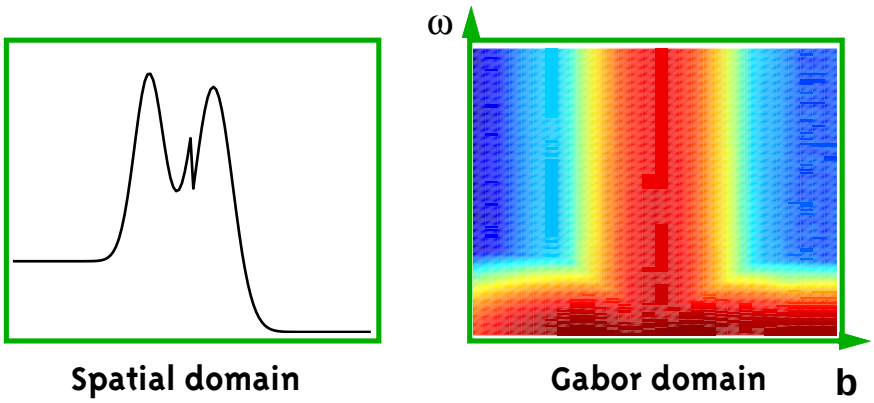
Gabor Transform



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Gabor Transform

Problems

- discrete version very difficult to find
- no fast transform
- fixed window size!

Solution

- large windows for low frequencies
- small windows for high frequencies

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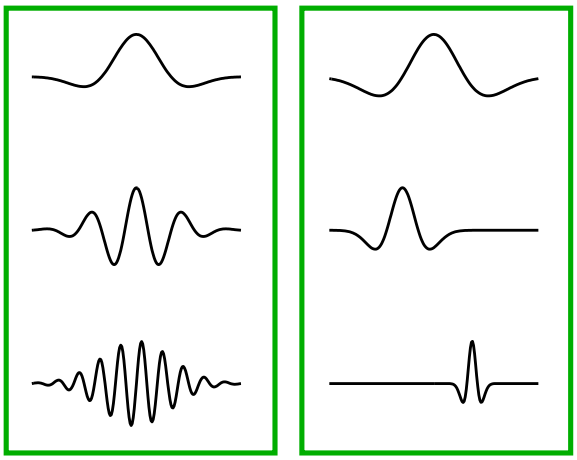
Gabor Transform



Gabor bases Wavelet bases

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Wavelets

Translates and dilates of one function

$$\psi\left(\frac{x-b}{a}\right)$$

Mother wavelet

- local in space
- local in frequency
  - smooth: no high frequencies
  - integral zero: no low frequencies

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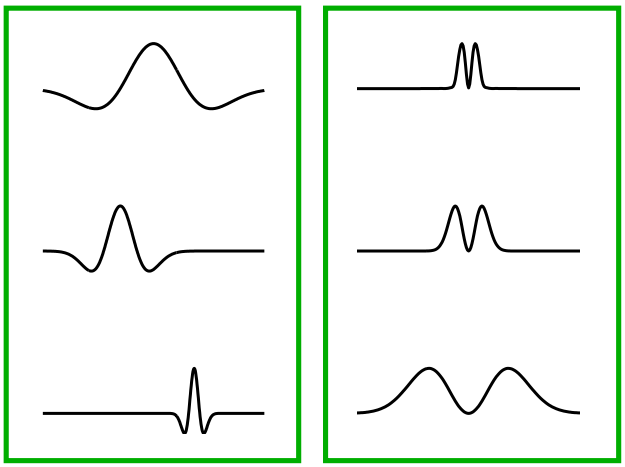
Wavelets



Wavelet bases Spectra

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Wavelet Transform

Find

- scale **a** at location **b**

function to analyze

$$F(a,b) = \int f(x)\psi\left(\frac{x-b}{a}\right)dx$$

Wavelet

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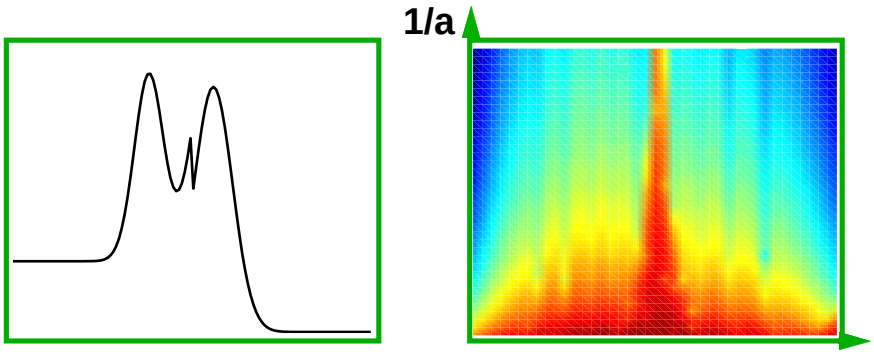
Wavelet Transform



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Summary

Fourier analysis
- global frequency properties

Picking out local phenomena
- windowed Fourier transform: Gabor

Wavelets
- window varies with frequency

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Making it Practical

A simple example
- Haar transform

Building more powerful transforms
- Lifting scheme

Generalizations
- making it work on general domains

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