deavor, low-bitrate compression remains a place where wavelet methods can prevail.

- *We should develop a wavelet image compression standard more general than just fingerprints.*
- *The idea of smoothness of functions should really be developed and applied as a tool for signal processing – incorporated into models of filtering and distortion. Donoho has already shown how smoothness and wavelets fit together for statistical denoising; similar attention should be given to the role of smoothness in image compression and restoration. Given the characterizations of Sobolev and Hölder smoothness in terms of wavelet coefficients, this problem area begs for the use of wavelet methods.*
- *There is a big need for wavelet constructions adapted to irregular two-dimensional sampling.*
- *Wavelets have just opened the door to the field of non-stationary, non-uniform, non-time invariant signal processing. This field is much larger than the field of time-invariant processing where the Fourier transform rules. It is like going from linear to non-linear operators. In this huge domain, wavelets are one important tools but many others are needed, wavelet packets, local cosine and may other transforms will come. Important orientations are: adaptive transforms, the use of other operators than convolution operators in signal processing applications, and development of stochastic models for non-stationary processes. Right now most things have been done on signals, but the same techniques can be used for numerical computations.*

"There have been too many pictures of Lena,
and too many bad wavelet sessions at meetings."
—Martin Vetterli, *Oberwolfach Meeting 1995.*

F. Dangers facing the field

- *There is too much media distortion concerning wavelets. Wavelets are oversold and in danger from their fashionability. We must say no to certain contracts. Claims by wavelet proponents, such as picture compression people, can not always be met. We should be more conservative in our claims.*
- *There is a danger of underselling wavelets! In the JPEG and MPEG standards, there are no wavelets. The DICOM competition will close soon – we should submit entries, or we will have to wait several years.*
- *There is a danger that wavelet people will split off from their previous specialties and start inbreeding. Wavelet people should preserve their roots.*
- *The fact that wavelets have a solid theoretical foundation will prevent them from burning out after 10 years.*

There does not seem to be a consensus amongst the answers here. Some people feel that wavelets are oversold, other feel exactly the opposite way.

VIII. Conclusion

We have tried to give an idea of what people feel wavelets are, why they work, and in which direction they are moving. Obviously this is only a snapshot taken from one point of view of a continuously evolving story and the only way to find out how it ends is to wait and see.

"Who controls the past,
ran the Party slogan,
controls the future:
who controls the present,
controls the past."
—George Orwell, *1984.*

IX. Acknowledgments

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problems. More than just a simple tool, wavelet ideas prompt new points of view. Some of the best ideas aren't written down. The big influence will come from the new generation researchers now growing up amidst wavelet ideas.

- *Wavelets have advanced our understanding of singularities. The singularity spectrum completely characterizes the complexity of the data data. Now we must go to an understanding of the underlying phenomena, to get the equation from the solution. Wavelets don't give all the answers but they force us to ask the right questions.*
- *Wavelets can be used to distinguish coherent versus incoherent parts of turbulence in fluid flow. They give some information but don't entirely solve the problem. Experiments provide high Reynolds' numbers with few measurements, while simulations provide many measurements but are restricted to low Reynolds' numbers.*
- *In Japan, wavelets resulted in a fruitful collaboration between academic research and industrial development.*
- *The results on regularity, approximation power, and wavelet design techniques, have led to significant developments in signal and image processing. For instance, some of the very best filter banks for image coding, including biorthogonal spline filter banks, have been designed from a wavelet point of view rather than the more traditional subband filtering point of view.*

"My dream is to solve problems,
with or without wavelets"
— Bruno Torresani, *Oberwolfach Meeting 1995.*

C. Theorems needed to advance the field

- *We need good theoretical models on how to distinguish signals from noise.*
- *We need to further study nonlinear multiscale methods in order to understand self-organizing multiscale systems with nonlinear relations between scales. Test cases are caricatures of turbulence, the stock market, or fractal piles of sand. There is no unifying theory at this moment, but maybe wavelets can provide one. We should not use linear multiscale models in these cases but instead look for nonlinear approximation with fast convergence. We need external measures to determine internal simplicity. The currently known tools are time-frequency analysis, smoothness, and* L_p *spaces.*
- *We need to understand simplicity in high-dimensional phenomena. We have tools and talent here to advance our understanding of such problems. Such problems will become more common as we collect more and more measurements for even simple physical systems.*
- *It is important to understand the right balance between simple building blocks and complex representations, or complex building blocks and simple representations. Only focusing on one is naive.*
- *We need to work on dictionaries as methods to representing complex objects in terms of simpler components.*
- *Many results in higher dimensions are still incomplete. Theoretical advances in higher dimensional signal approximation bounds, regularity, design techniques, would be very useful in image processing applications. There is room for substantial improvement of the current state of the art.*

"Splines approximate; wavelets show internal structure."
— Wolfgang Dahmen, *Oberwolfach Meeting 1995.*

D. What kind of wavelets do we still need?

- *We would like to have isotropic, compactly supported, and orthogonal wavelets, but unfortunately they don't exist. Multi-wavelets can provide an answer here.*
- *Wavelets with custom-design properties, like wavelets on manifolds and wavelets adapted to irregular samples are on their way.*
- *We need to work on spanning the gap between eigenfunctions and wavelets.*

  There seems to be a debate on the relationship between splines and wavelets. Some people feel that splines and spline based wavelets will be able to span the gap between eigenfunctions and wavelets. For example, splines already are exact solutions to variational problems. Other people feel that there is no fundamental difference between splines and (non spline based) wavelets and that each exact spline solution has a corresponding almost-solution in wavelets.

E. Problems not sufficiently explored with wavelets

- *Prediction: The stock market, earthquakes, weather. We should use wavelets to build models based on previously observed data.*
- *Physics: We can use wavelets to define the scale of the observer. Wavelet techniques can be applied to quantum physics, atoms, and lasers.*
- *Scientific computing and in particular computational chemistry. We need to look for problems that need multiscale and non-linear approximation methods.*
- *Wavelets and filter banks should be systematically applied to problems in communications: for example transmultiplexers to map many different signals onto one channel. Wavelets and filter banks provide a broad, rich class of new transforms with which to do this transmultiplexing, offering the familiar advantages of time localization and frequency selectivity. Because wavelets were initially developed in conversation with the subband coding community, these applications application has gotten less attention. However, because of the large global telecommunications market, telecommunication applications of wavelets could be the sleeper that surprises everybody.*
- *Really make wavelet methods work for video compression. While MPEG's success and proliferation make medium-bitrate wavelet video a purely academic en-*

Littlewood-Paley theory, non-linear approximation theory, fractals and self-similarity, splines, filter banks, sampling theory, Calderon-Zygmund theory, pyramid algorithms in image processing, ... One small example: adaptive methods for solving PDEs again corresponds to non-linear approximation.

Nobody will dispute the usefulness of considering a general framework to study these ideas. This common framework, somehow, ended up being called "wavelets." Whether this is the right name is open for discussion.

The most important point is that by putting different ideas into a common framework new insights are gained. For example, non-linear approximation has been studied for a long time from a theoretical point of view. Wavelet constructions now provide a computational framework to go along with this theory. On the other hand the transform behind subband coding already was a fast wavelet transform, but some of the theoretical background was missing. Wavelets thus form a new bridge between the theory and practice bridgeheads.

When faced with a problem from a particular area where multiresolution and non-linear approximation seems useful, wavelets provide a flexible prototyping environment that comes with fast computational algorithms and solid theoretical foundation. When studying a particular problem using wavelets different situations can occur:

- The wavelet based algorithms outperform the existing ones. In this case the wavelet machinery provides new insights both theoretically and practically.
- The wavelet based algorithms do just as well as the existing ones. In this case the wavelet theory might help proving this behavior or simplifying some of the existing proofs.
- The wavelet based algorithms are inferior to the existing ones. This could mean that wavelets are not the right answer for this problem or that the expertise in this particular area surpasses the understanding brought by wavelets. In the latter case, the wavelet machinery can be enriched with these new ideas.

In the past all three situations have occurred. In my opinion, the future of wavelets lies in further exploring the interplay and expertise interchange between wavelets and other research fields.

> *"All questions are insightful and carefully researched. Anybody who claims otherwise is itching for a fight"*
> —Martin Feldman, *"What do you know?" NPR.*

VI. Questionnaire

The following questionnaire was distributed to a number of researchers involved with wavelets.

1. In your opinion, which are the most promising developments in wavelets?
2. Where do you feel the field is moving?
3. What kind of theoretical developments (theorems) are needed to further push forward the field?
4. What kind of wavelet functions should be further be sought after?
5. Which application domains have not yet been sufficiently explored with wavelets?
6. Where will wavelets be in 10 years from today?
7. Have wavelets kept their promises?
8. Are there any dangers threatening the wavelet field?

VII. Answers

This section contains the digested version of the answers. It is organized per topic and each topic consists of a list of paragraphs. Each paragraph combines similar answers, but, as you will notice, sometimes paragraphs contradict each other.

> *"Doublethink means the power of holding two contradictory beliefs in one's mind simultaneously, and accepting both of them"*
> —George Orwell, *1984.*

A. Wavelets and PDEs

- *On the practical side significant progress has been made in boundary element methods since the Beylkin-Coifman-Rokhlin paper. Wavelets seem to be more general than multi-pole methods. We still need fast quadrature formulas for the discretization of integral operators.*
- *In case of elliptic PDEs the existing algorithms are good. Wavelets will not necessarily outperform them, but they could be equal. Multigrid is very efficient, especially on the simple problems done so far with wavelets. We should combine wavelet and finite element methods. For example, we can now prove that finite element method codes work using the norm equivalence of unconditional wavelet expansions.*
- *Classical wavelets seems to be good in computing low frequency scattering and antenna problems. However they fail in case of very high frequencies. Additional engineering expertise is needed to solve the problem. High frequency oscillatory integral kernels need local cosines, not classical wavelets.*
- *In the context of building solvers for integral equations, adaptive methods are crucial. There is no point in first building a full $N \times N$ matrix at a cost of $O(N^2)$, and then using wavelets to invert the matrix in $O(N)$ steps. One should immediately build the compressed matrix in the wavelet basis. This approach needs of fast and accurate error estimators.*

B. Interesting Recent Wavelet Developments

- *Denoising has both opened up other fields and imported techniques such as dictionaries and nonlinear approximation from other fields. Nonlinear approximation, smoothing, and reduction to small optimization problems, are real achievements.*
- *Wavelets have had a big psychological impact. People from many different areas became interested in time-frequency and time-scale transforms. There has been a revolution in signal processing. There is less specialization, and the subject is now opened to new*

well. Such building blocks will be able to reveal the internal correlation structure of the data sets. This should result in powerful approximation qualities: only a small number of building blocks should already provide an accurate approximation of the data.

We can immediately relate this back to the wavelet properties. Assume we want to approximate a function f with a function f_M which is a linear combination of M wavelets. The question is: which M wavelets out of a possibly infinite set should we pick and what coefficients should we given them? Ideally we want to choose f_M so that $f - f_M$ is minimal in the norm of $\mathcal{F}$ for all possible choices of M wavelets and M coefficients. This would be optimal, we simplify things by using the equivalent norm in C. This is not optimal, but we can be off only by a factor of at most B/A. Given that the norm in C only depends on the absolute values of the coefficient, we find the best approximation in the norm of C by simply choosing the M largest (in absolute value) coefficients, i.e., we let

$$f_M = \sum_{\lambda \in \Lambda_M} c_\lambda \, \psi_\lambda,$$

where Λ_M contains the indices of the M largest wavelet coefficients. Since the index set Λ_M depends on the function f, the approximation is non-linear: $(f+g)_M$ is not equal to $f_M + g_M$. Indeed, the latter can contain up to $2M$ wavelet terms. If M goes to infinity, f_M will converge to f in the norm of $\mathcal{F}$. In order to quantify the approximation properties of f_M, i.e., to verify if we captured the internal correlation structure of the data, we look at the speed of convergence as we add more terms. This is given by the largest α for which

$$\|f - f_M\| = O(M^{-\alpha}). \qquad (2)$$

If α is large, the essential information contained in a function is captured by a small fraction of the wavelet coefficients. This is the key to applications. The question on how to find α has been studied extensively in the area of non-linear approximation and smoothness spaces. The main result says that if $\mathcal{F}$ is a Besov space of smoothness α, then (2) holds. We do not intend to give a precise mathematical formulation of Besov spaces here, but rather try to give some intuition on why they are important. For example, if a function belongs to C^α, i.e, if this function is α times continuously differentiable, and its α-th derivative belongs to L_2, then (2) holds. However, for this kind of functions traditional linear approximation methods based on the Fourier transform also give convergence speeds like (2). But C^α functions are not very good models for real life signals. A much better model, e.g. for images, are functions which are *piecewise* smooth. Indeed images have discontinuities (edges), with smooth regions in between. These functions still belong to Besov spaces with high (typically $\alpha \geqslant 2$) smoothness index. Besov spaces thus provide appropriate models for real life signals. For piecewise smooth functions, linear, Fourier-based methods give very slow convergence (e.g, $\alpha = 1$), while non-linear wavelet-based method still exhibit the fast convergence (e.g, $\alpha \geqslant 2$). This property results in the fact that, quoted from Donoho, "*wavelets are optimal bases for compressing, estimating, and recovering functions in $\mathcal{F}$.*"

IV. Instances of wavelets

There are several instances of functions that exhibit the above mentioned properties. As Jelena and Ingrid worded it, we follow the "democratic" approach and refer to all of them as wavelets. Some examples are:

- **Dyadic translates and dilates of one function:** These are the classical wavelets. They naturally connect with multiresolution analysis and subband coding. For more information we refer to the article by Albert Cohen and Jelena Kovačević.
- **Wavelet packets:** This is an extension of the classical wavelets which yields basis functions with better frequency localization at the cost of a slightly more expensive transform. For more information check the article by Nicolaj Hess-Nielsen and Victor Wickerhauser.
- **Local trigonometric bases:** The main idea is to work with cosines and sines defined on finite intervals combined with a simple, but very powerful way to smoothly join the basis functions at the endpoints. For more information we again refer to the article by Nikolaj Hess-Nielsen and Victor Wickerhauser.
- **Multi-wavelets:** The idea is not to use *one* fixed function to translate and dilate but rather a finite number. This way one can obtain combinations of useful properties which were impossible with classical wavelets.
- **Second generation wavelets:** Here one entirely abandons the idea of translation and dilation. This gives extra flexibility which can be used to construct wavelets adapted to irregular samples, weights, or manifolds. For more information, we refer to the paper by Peter Schröder.

We should point out that the Fourier transform does *not* fit into this picture, as the basis functions do not have space localization and the powerful non-linear approximation properties fail.

> *"It's the place where my prediction from the sixties finally came true: 'In the future everybody will be famous for fifteen minutes.' I am bored with that line. I never use it anymore. My new line is, 'In fifteen minutes, everybody will be famous.'"*
> —Andy Warhol, *Exposures (1979) 'Studio 54'.*

V. Are wavelets fundamentally new?

As we mentioned earlier, research can be thought of as a continuous growing fractal which often folds back onto itself. This folding back definitely occurred on several occasions in the wavelet field. We point out that many of the idea and properties mentioned in the previous sections have been circulating in different areas. We mention here a few: subband coding, subdivision for CAGD, multigrid for the numerical solution of partial differential equations,

A. Wavelets are building blocks for general functions.

We want to express a general function f of $\mathcal{F}$ as an infinite series of wavelets. Thus a coefficient sequence $c = \{c_\lambda \mid \lambda \in \Lambda\}$ has to exist so that

$$f = \sum_\lambda c_\lambda \, \psi_\lambda. \tag{1}$$

We call this a *wavelet series* and assume it converges in the norm of $\mathcal{F}$. We require that the converge is unconditional, i.e., independent of the order of summation. Consider the space C which contains all coefficient sequences $c \in C$, for which the corresponding function f belongs to $\mathcal{F}$:

$$C = \{c \mid \sum_\lambda c_\lambda \, \psi_\lambda \in \mathcal{F}\}.$$

We want an explicit characterization of this space, i.e., we want to tell whether a coefficient sequence c belongs to C without having to synthesize f. This is done through a norm on C so that $c \in C$ if and only if $\|c\|_C < \infty$. The norms of C and $\mathcal{F}$ should be equivalent in the sense that positive constants A and B exist so that

$$A \, \|c\|_C \;\leqslant\; \|f\|_{\mathcal{F}} \;\leqslant\; B \, \|c\|_C.$$

The fact that the norm of C depends only on the absolute value $|c_\lambda|$ of the coefficients is related to the unconditional convergence of the series. The convergence thus cannot depend on cancelations due to alternating signs of the coefficient sequence.

How do we now find the coefficients, given a function f? They are linear functionals of f written as

$$c_\lambda = \langle \, \widetilde{\psi}_\lambda, f \, \rangle \, ,$$

where $\langle \cdot, \cdot \rangle$ is the appropriate pairing. The $\widetilde{\psi}_\lambda$ are the dual wavelets. If $\mathcal{F}$ is a Hilbert space, the dual wavelets belong to $\mathcal{F}$ as well, otherwise they belong to the dual of $\mathcal{F}$. There are different instances of functions with these properties. The nicest somehow is an orthonormal basis in which case the coefficients are unique, the dual and primal wavelets coincide, and the discrete (C) and continuous norms ($\mathcal{F}$) are equal. In case of an unconditional basis, the coefficients are still unique but the norms are only equivalent, and in the case of a frame, the coefficients are not unique but can be chosen so that the norm equivalence holds.

B. Wavelets have space-frequency localization.

Consider the case of the real line. Locality in space implies that most of the energy of a wavelet is restricted to a finite interval. Ideally the function is exactly zero outside the finite interval: a so-called compactly supported function. In general we want fast, e.g. inverse polynomial or exponential, decay away from the center of mass of the function. Frequency localization simply means that the Fourier transform of a wavelet is localized, i.e., a wavelet mostly contains frequencies from a certain frequency band. The Heisenberg uncertainty principle puts a lower bound on the product of space and frequency variance. The decay towards high frequencies corresponds to the smoothness of the function. The smoother the function, the faster the decay. If the decay is exponential, the function is infinitely many times differentiable. The decay towards low frequencies corresponds to the number of vanishing moments of the wavelet. A wavelet ψ_λ has N vanishing moments in case

$$\int_{\mathbf{R}} x^p \, \psi_\lambda \, dx = 0,$$

for $0 \leqslant p < N$. Thinking of "frequency localization" in terms of smoothness and vanishing moments, allows us to generalize this notion to setting where no Fourier transform is available, e.g. manifolds.

C. Wavelets have fast transform algorithms.

It has to be easy to implement wavelet functions on a computer. Typically we want an algorithm with linear or linear-logarithmic complexity to pass between a function f and its wavelet coefficients c. Such an algorithm is referred to as a "fast wavelet transform."

Fast wavelet transforms are often obtained through *multiresolution analysis*. The idea is to approximate the function f at different levels of resolution. Think for example of representing an image with fewer and fewer pixels. The wavelet coefficients can then by found as the additional detail needed to go from a coarser to a finer approximation. Combining all detail coefficients together then leads to the wavelet coefficients c.

We should point out that these three properties are not unrelated. For example, if the wavelet basis is orthogonal, then the coefficients are simply given as the inner product of f with the basis function, which greatly simplifies the transform algorithm. On the other hand, if the wavelets are well separated in the time-frequency domain, it is quite likely they form an unconditional basis.

> *"In research the horizon recedes as we advance,*
> *and is no nearer at sixty than it was at twenty.*
> *As the power of endurance weakens with age, the*
> *urgency of pursuit grows more intense . . .*
> *And research is always incomplete"*
> —Mark Pattison, *(1875).*

III. Why do wavelets work?

Why do we want these properties? Most of the data which we encounter in real life is not totally random but has a certain correlation structure. Think for example of audio signals, images, solutions of differential equations, time series, etc. The correlation structure of many of these signals is similar. They have some correlation in space (or time), but the correlation is local. For example, neighboring pixels in an image are highly correlated but ones that are far from each other are uncorrelated. Similarly there is some correlation in frequency, but again it is local. Indeed, the spectrum of many signals has a band structure.

This motivates approximating these data sets with building blocks that have space and frequency localization as

Wavelets: What Next?

Wim Sweldens

Abstract—**In this concluding article, we want to look ahead and see what the future can bring to wavelet research. We try to find a common denominator for "wavelets" and identify promising research directions and challenging problems.**

Keywords— **Wavelet, Future.**

"Predicting is hard, especially about the future."
—Victor Borge, *quoted by Philip Kotler.*

I. Introduction

As the articles in this special issue show, wavelets have been successfully applied in a wide variety of research areas. These papers prove that wavelets provide a common framework to study problems that at first sight seem unrelated. Part of the power of wavelets comes from the fact that they lie at the crossroads of a wide variety of research areas.

As the title suggests, this paper consists of two parts: *Wavelets* and *What Next*. In the first part, we discuss what is currently meant by a *wavelet*. We do so by outlining a common denominator for various developments which have been called *wavelet*. In the second part, we want to look ahead and ask ourselves the question: "What next?" We point out in which direction the wavelet research front is moving and identify challenging problems.

In order to gather information for this paper, we sent a questionnaire (see Section VI) to various researchers, both inside and outside the wavelet field. More particularly these questions were addressed in a round table discussion called by Marie Farge and Ingrid Daubechies at the wavelet Oberwolfach meeting held in August 1995. The second part is based upon the responses to this questionnaire. Sometimes literal quotes will be included, at other places we present the answers in digested form.

Writing this paper is equally hard as predicting the future, i.e., virtually impossible. Chris Heil worded this nicely in his response: *"If anybody actually knew the future of wavelets well enough, they probably would be writing a paper about it and ..."* We start out by tuning down our ambition. We do not pretend to foretell the future but rather to discuss *current* exciting developments and identify powerful research *directions*.

The previous articles of this issue discuss the application of wavelets in particular fields and are written by leading experts in those fields. Each of them already point to a particular exciting direction in which the wavelet field is moving. Here we do not address each of these individual fields but rather give an overview on a coarser (sic) level.

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Like the article on the history on wavelets, this article on their future only contain a *personal* view. Realize that it is written from the perspective of a junior researcher who does not even know the time before wavelets. The future of a research field can be a very touchy subject. I have tried to lighten up the tone of the whole article. There is no need to take these writings too seriously: let us all relax and enjoy discussing current issues in wavelet research ;-)

In the light of this, I decided not to include any references. However, it should be crystal clear that I do not claim authorship of any of the ideas in this paper. It is simply a write-up of understanding that has been *floating around in the community*. So no credit is given, nor taken.

"If you steal from one author, it's plagiarism; if you steal from many, it's research"
—Wilson Mizner, *The Legendary Mizners (1953).*

II. What are wavelets?

In this section we give an idea of what makes a function a wavelet and why wavelets are desirable in certain applications. Given that the wavelet fields keeps growing, the definition of a wavelet continuously changes. Therefore it is almost impossible to rigorously define a wavelet. As in most areas of research, the wavelet knowledge front advances like an infinite dimensional fractal, sometimes taking off in isolated directions but also many times folding back onto itself. Finding a definition of a wavelet is like approximating this fractal with a ball. The "minimal" solution is a ball with a small radius which fits in the interior of the fractal. But this leads to a definition that contains only the very core material and leaves out most of the recent and very exciting developments at the forefront. The "maximal" solution is fitting the fractal in a ball. This results in a definition that includes almost any function.

Therefore we step away from the idea of giving a rigorous definition and allow ourselves the freedom to make "fuzzy" statements. We urge the reader to bear with us and not judge this by the usual scientific standards. For precise mathematical statements, clear definitions, rigorous material, and correct scientific results, we refer to the earlier articles in this issue. For vague descriptions, flaky statements, an exposition which sweeps most details under the carpet, lots of hand waving, etc., keep reading.

We denote a wavelet as $\psi_\lambda(x)$ where x belongs to the (undefined) spacial domain X, ψ_λ belongs to a (undefined) class $\mathcal{F}$ of functions, and λ belongs to an (undefined) index set Λ. Think for example of X as the real line, $\mathcal{F}$ as $L_2(\mathbf{R})$, Λ as $\mathbf{Z}^2$ with $\lambda = (j, l)$, and $\psi_\lambda(x)$ as $2^{j/2}\psi(2^j x - l)$. We will refer to $\Psi = \{\psi_\lambda \mid \lambda \in \Lambda\}$ as the wavelet basis. In order to justify calling Ψ a wavelet basis, we expect the following three properties.